response to ARK#2 12/03/99

[**RMK SAYS**]
Well it looks as if some progress has been made
from here ******************************
On 2 Dec 99, at 14:28, RKiehn2352@aol.com wrote:
> IF I remember my training the Lorentz force is defined as
> E + V x B on a unit charge moving with V.
> DO YOU DISAGREE OR AGREE.
agree
> It appears to me that W = i(V)dA generates that formalism.
> E + V x B
> DO YOU AGREE OR DISAGREE
Yes, this is how Lorentz force is always represented
in a relativistic form - assumingthe fourh component of
four-velocity being = 1.
> There are no derivatives of V in the expression for W
> DO YOU AGREE OR DISAGREE
agree

to here*********************************

> The definition of W also indicates that
> F(Lorentz) dot V = E.V, for any V,
> the power theorem that I learned long ago
> as being useful in applied electromagnetism.
> **
I do not know what is your notation. It becomes
now confusing.
1) What is F(Lorentz)?
Is F(Lorentz) = E + V x B
as above?
What is dot V? How you calculate dot V?
Is it the same V as in the Lorentz force?
Is time derivative computed along trajectory

of the charged particle? If so, then I disagree.

Or is dot a scalar product symbol? If it is

a scalr product, yes

F(Lorentz).V = (E + V x B).V = E.V because VxB is perpendicular to V

[RMK SAYS]

(All of this notation is spelled out in

http://www22.pair.com/csdc/pdf/maxwell.pdf. but you "have better sources")

In terms of the better notation available with pdf files, consider:

the set of independent variables with symbols $[x, y, z, t]$

and a vector field defined as

$$\lambda V = \lambda(x, y, z, t)[V^x(x, y, z, t), V^y(x, y, z, t), Vz(x, y, z, t), 1] = \lambda[\mathbf{V},1] \quad (1)$$

such that $i(V)(dx^k - V^k dt) = 0$,

and *any* 1-form

$$A = A_x(x, y, z, t)dx + A_y x, y, z, t)dy + A_z x, y, z, t)dz - \phi x, y, z, t)dt.$$

Guided by the format of the Lie derivative with respect to V acting on A, where $L_{(X)}dA = i(X)dA + d(i(X)A)$, define the 1-form W as (the first -not exact- term in the Lie derivative formula):

$$\begin{aligned} W &= i(V)dA \;\; which\, when\, evaluated\, on\, \{x, y, z, t\}\, becomes: \\ &= \sum_{k=1}^{3}\{-grad\phi - \partial\mathbf{A}/\partial t + \mathbf{V}\times curl\mathbf{A}\}_k dx^k - \{(-grad\phi - \partial\mathbf{A}/\partial t)\cdot\mathbf{V}\}dt \end{aligned}$$

If $i(V)dA = 0$ it is convential to describe such vector fields as *extremal* fields. If $i(V)A = 0$, the field is described as an *associated* vector field. If both equations vanish, the vector is said to be a *characteristic* vector field for the 1-form A (see P. Libermann, Klein,..)

As $i(V)i(V)dA = i(V)W = 0$, then it follows that

$$\sum_{k=1}^{3}\{-grad\phi - \partial\mathbf{A}/\partial t + \mathbf{V}\times curl\mathbf{A}\}_k V^k - \{(-grad\phi - \partial\mathbf{A}/\partial t)\cdot\mathbf{V}\} = 0. \quad (2)$$

or

$$\mathbf{F}_L \bullet \mathbf{V} - \{Power\} = 0. \tag{3}$$

However, the statement is true for *any* V, and *any* set of functions that define the 1-form, A, in any number of dimensions. On $\{x, y, z, t\}$, the "Lorentz Force" $\mathbf{F}_L$ is defined by the symbols

$$\mathbf{F}_L = \{-grad\phi - \partial\mathbf{A}/\partial t + \mathbf{V}\times curl\mathbf{A}\} \tag{4}$$

and is called the "Lorentz Force" because if $\mathbf{A}$ is interpreted as the vector potential per unit charge, and ϕ is the scalar potential, as subsumed by classical electromagnetic theory, then the Force has the familiar (agreed to) format,

$$\mathbf{F}_L = \{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}. \tag{5}$$

which is the format of the Lorentz force law. However, in fluid dynamics, where the vector potential is presumed to be the velocity field itself ($\mathbf{A} \Rightarrow \mathbf{V}$), the same formula is still valid for the "force" and the term $\mathbf{V} \times \mathbf{B} \Rightarrow \mathbf{V}\times curl\mathbf{V}$ is called the "Magnus force".

In the symbolism of EM, the Power per unit charge becomes

$$Power = \mathbf{E} \bullet \mathbf{V} \tag{6}$$

Note this is a "universal" result independent of the interpretations placed upon the functions that make up the construction. The basic idea that $i(V)i(V)dA = i(V)W = 0$, is valid in every coordinate system.

****

> I believe you have something to reconsider.

> Perhaps your "sources" need reinvestigation.

Which sources?

[**RMK SAYS**]

You told me you had better sources (than the reference I suggested, which was:

http://www22.pair.com/csdc/pdf/maxwell.pdf).

I do not know "your sources"

****

> The same method applied to the
> 1-form
> A = v dot dr - (v^2+phi)dt
> will generate the Euler fluid equations.
> **

I do not know what you are talking about now.
What "applies" THE SAME WAY?

[RMK SAYS]

As mentioned above, the formula for the work 1-form, $W = i(V)dA$, obtained as the non-exact component of the Lie derivative, can be used to generate equations of motion for systems other than electromagnetism. With a change of notation, ($\mathbf{A} \Rightarrow \mathbf{v}$, $\phi \Rightarrow \mathbf{v} \bullet \mathbf{v}/2 + P/\rho$) the extremal solutions for "zero" virtual work, $W = i(V)dA$, satisfy the equation

$$W = \partial\mathbf{v}/\partial t + grad(\mathbf{v} \bullet \mathbf{v}/2) - \mathbf{v} \times curl\mathbf{v} = -grad(P/\rho \tag{7}$$

which many folks in the applied world recognize as the Euler equation for a barotropic fluid.

(Same universal concept, different notation, different physical application.)

****

> Searching for solutions where
> d(i(V)dA) <> 0
> will generate Navier Stokes equations.
> **

What is the relation of this to the first part?
There is no relation. You are now consider
a different problem, and you do not even
try to explain what is this new problem.

[RMK SAYS]

Much of the detail is in the maxwell.pdf that evidently you think is not worthy of reading. (Tant pis pour vous) (But as you said you have better sources.)

However, the relation is that as $W = d\Theta$ defines all Hamiltonian systems (Cartan's theorem), then situations where $dW \neq 0$ may be of interest to define equations of motion for systems that are not conservative. Indeed that is the case, for it is possible to deduce the Navier-Stokes equations from such methods.

As you have better sources use then, otherwise go to the library and look up: R. M. Kiehn "Intrinsic Hydrodynamics with Applications to Space-Time Fluids" Int. J. of Engng Sci. VOl 13, pp 941-949 (1975). This article is one of the few of my articles not available as a pdf download on my website.

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> The equations of motion come from W = i(V)dA

> in the calculus of variations and in forms language.

> **

You have to explain what you mean. I have no idea.

> Cartan demonstrated the any form A

> such that i(V)dA = d(theta)

> had equations of motion such that the V were of the

> Hamiltonian format. (an iff theorem)

What equation of motions? In which framework?

In which context?

> His proof (see Lessons on Integral Invariants) was deduced

> by searching for evolutionary fields V

> such the the closed integral of A was an invariant.

Proof of what? Can you state the exact theorem you are now talking about? And how it relates to Lorentz force?

> HEnce:

> Lie derivative of closed integral of A = 0 is necessary and sufficient.

> **

What is the stament? While the first part, where you were asking me, whether I agree or not, was clear (and I agreed, but it was irrelevant), now you are using vague terms.

[**RMK SAYS**]

The idea that $L_{(X)}A = W + dU$ with $W = i(X)dA$ and $U = i(X)A$ is universal for any set of differentiable functions defined on any set of local coordinates. Cartan's theorem basically says that all Vector fields, X,with a scaling function λ, such that

$$L_{(\lambda X)} \int_{closed\_cycle} A = 0 \tag{8}$$

for any non-zero λ, have a Hamiltonian generating function. The requirements imply that

$$W = i(X)dA = d\Theta \tag{9}$$

such that $dW = 0$. The concept works in any dimension for any set of functions that define the 1-form and the vector field. Hence, any problem that can be cast into the form $W = i(X)dA = d\Theta$ is equivalent to a problem in Hamiltonian mechanics. The Hamiltonian equations are usually accepted as definitions of a (very large) class of "equations" of motion.

To demonstrate the case choose notation on a contact manifold of dimension 2n+1, such that

$$A = pdq - H(p, q, t)dt \quad and \quad X = [\dot{p}, \dot{q}, 1] \tag{10}$$

Note that at this stage the functions $\dot{p}, \dot{q}$ are **not** identified as total derivatives with respect to time. Then in the odd dimensional space choose $\Theta = cons\tan t$ such that X is the unique extremal field. Then

$$\begin{aligned} W &= i(X)dA = i(X)\{dp\hat{}dq - dH(p, q, t)\hat{}dt = 0 \\ &= \dot{p}dq - \dot{q}dp + (\partial H/\partial q)\dot{q}dt + (\partial H/\partial p)\dot{p}dt + dH \\ &= \{\dot{p} - (\partial H/\partial q)\}dq + \{-\dot{q} + \partial H/\partial p\}dp + \{(\partial H/\partial q)\dot{q} + (\partial H/\partial p)\dot{p} + \partial H/\partial t\}dt \end{aligned}$$

The system is satisfied when the functions that define the vector field are computed from the formulas:

$$\{\dot{p} - (\partial H/\partial q)\} = 0 \quad and \quad \{-\dot{q} + \partial H/\partial p\} = 0 \tag{11}$$

These are the defining equations for the Hamiltonian extremal process. If now the anholonomic constraints

$$dq - \dot{q}dt = 0 \quad and \quad dp - \dot{p}dt = 0 \tag{12}$$

are also applied, the problem reduces to the usual "particle" interpretation of mechanics. However, these kinematic constraints are not necessary and in particular are **not** applicable to fluids where anholonomic fluctuations can be non-neglible.

See

http://www22.pair.com/csdc/pdf/topturb.pdf

http://www22.pair.com/csdc/pdf/topevol.pdf

http://www22.pair.com/csdc/pdf/irrev1.pdf

http://www22.pair.com/csdc/pdf/photon5.pdf

http://www22.pair.com/csdc/pdf/topstruc.pdf

http://www22.pair.com/csdc/pdf/jhelical6.pdf

no pdf file available: "Topological Defects, Coherent Structures and Turbulence in Terms of Cartan's Theory of Differential Topology" in "Developments in Theoretical and Applied Mathematics" Proceedings of the SECTAM XVI conference, B. N. Antar, R. Engels, A.A. Prinaris and T. H. Moulden, Editors, The University of Tennessee Space Institute, Tullahoma, TN 37388 USA. (1992) p. III.IV.2

Again the mechanism is independent of the symbols used. Vector fields satisfying Cartan's criteria have a Hamiltonian generator. The same can be done for the even dimensional 2n+2 symplectic space. Cartan's concept says that all Hamiltonian systems belong to this topological equivalence class characterized by the anholonomic constraint that the 1-form of Work be exact., and every differential form A for which the Work 1-form $W = i(V)dA$ is exact admits processes of evolution of the Hamiltonian type.

If the format is as above for EM theory, and if

$$W = i(V)dA = d\Theta, \tag{13}$$

then the same analysis holds, and there is a generator that produces the "Hamiltonian flow" for every such system of vector and scalar potentials. Under these circumstances the "Lorentz Force" is a gradient field with zero curl.

****

> In that case the perfect differentials in the definition of the Lie

> derivative do not contribute, for the closed integral of a perfect

> differential is zero.

There are no derivatives of V in the Lorentz foce. Therefore

Lie derivative cannot help you to get it.

[**RMK SAYS**]

Baloney, the Lie derivative focuses attention on the 1-form of Work, which

does not contain derivatives of the velocity, and has the format of the Lorentz force.

****

> NOte also that dA is an evolutionary invariant of a HAmiltonian system

> as L(V)dA = dd(theta) = 0.

What are you talking about and how it relates to Lorentz force?

There are no derivatives of V in the Lorentz foce. Therefore

Lie derivative cannot help you to get it.

[**RMK SAYS**]

Baloney: You agreed that W does represent the Lorentz force and it comes directly from Cartan's Magic formula for the Lie derivative. (Marsden gave the name Magic formula)

There are no derivatives of V in the work 1-form, W, but the Lie derivative did help in separating the non-exact work 1-form, W (with its Lorentz force format - free of charge) from the cohomology component dU.

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> A restatement of the Helmhoiltz theorem (and related to gauge invariance)

> which leads to the invariance of the even dimensional Poincare integrals.

> ***

> Another fact of cohomology.

> HOW CAN I BE MORE CLEAR.

By keeping to the standard of clarity. You were doing well

before the line **********************************

But you gave up after ************************* and

what was after is either irrelevant or incomprehensible.

There is no derivative of V in the formula for the

Lorentz force. There are derivatives of V in the formula

for Lie derivative of vector potential. Therefore othey are two

essentially different things. You can't put derivatives ov V

under the rug. They will call from there:

"WE ARE HERE!!!!!!!!!!" They will go after you, they will haunt you, and calling names Cartan and Euler (and even Hamilton) will not help. They will not disappear!

[RMK SAYS]

Recall that I did not claim the the Lorentz force was equal to the Lie derivative.!

What is your hang up here? Mu claim is that the Lorentz force comes from the non-exact component of the Lie derivative. That component does NOT contain derivatives of V.

You are putting your own spin on my statements.

ALSO

A manipulation of adding and subtracting (as occurs in Cartan's magic formula) is similar to the "integration" by parts trick in the calculus of variations. The perfect differential (which does contains derivatives of V) has zero contribution to any integral whose integration domain is a closed cycle. Hence the only contributions to the evolution of a closed integral

$$L_{(\lambda V)} \int_{cycle} A = \int_{cycle} W + \int_{cycle} dU = \int_{cycle} W + 0 = \int_{cycle} W \qquad (14)$$

come from the Work 1-form (and thats the part that contains the Lorentz Force!!!!!). It is this formula that led Cartan to his concept of a tube of trajectories, where the integration chain around the tube of trajectories could be deformed and yet the integral remains the same when W is exact ($W = d\Theta$). The integral is therefor a deformation invariant (as $\lambda(x, y, z, t)$ distorts the points of the integration chain along the trajectories) when $L_{(\lambda V)} \int_{cycle} A = 0$. This explicitly demonstrates part of the topological properties of the method.

The perfect differential part of the cohomology statement (the dU) can be ignored in many situations. What is the cohomology statement produced by the Lie derivative ? It is

$$W + dU = Q \qquad (15)$$

Engineers recognize this formula as the first law of thermodynamics. The Cartan magic formula is extraordinary for it makes an explicit link to dynamics and thermodynamics without the use of statistics. One of my latest results is that the method can be used determine thermodynamic irreversibility for processes defined by vector fields acting on physical systems defined by Action 1-forms.

Another trick used in the calculus of variations (to derive equations of motion) is to say that the components of U vanish on the boundary. So indeed these are constrained systems.

Indeed the Voices you refer to do call out. They say there is something different between the concepts of Heat and Work. But that does not alter the fact that that the Lorentz force law exists from the definition of $W = i(V)dA$, which was deduced from the Cartan definition of the Lie derivative. What your "voices" say is that those processes constrained to be Hamiltonian (such that $dW = ddU = 0$) only approximate the real world, where it appears that everything is irreversible. Processes for which the "Lorentz force" are without curl are never thermodynamically irreversible. A necessary but not sufficient condition for irreversibility is that $dW \neq 0$. How do you study irreversible processes? You study those cases where the Lorentz force is not closed – where the Helmhotz theorem is not valid ($L_{(\lambda V)}(dA) = dW \neq 0$). Not that the Failure of the Helmholtz theorem depends of the fact that the Lorentz force has a "curl".

The details of these notions are well layed out in the my articles on the web or in the literature, and do not need repeating here. You might take a look at them.

see

http://www22.pair.com/csdc/pdf/copen5.pdf

http://www22.pair.com/csdc/pdf/irevtorsd.pdf

http://www22.pair.com/csdc/pdf/short2.pdf

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IN SUMMARY

The Lorentz force IS NOT equal to the Lie derivative acting on forms.

The Lorentz force is deduced from the non-exact component $W = i(V)dA$ of the Lie derivative.

HOW CAN I BE MORE CLEAR?

regards

RMK