response to ARK

[from the original email]
> Hi
> I thought ARK understood the Action of the Lie derivative on forms.
> **
[ARK SAID]
He does
> For details see
> http://www22.pair.com/pdf/maxwell.pdf
> ***
[ARK SAID]
He has better sources.
[RMK says]
Evidently your sources are different from mine.
But it is evident that you did not bother to look
(Pity – For if you had looked, this conversation would have been moot)
> in email notation.
> on (x,y,z,t)
> Let A = Axdx+Aydy+Azdz - phi dt.
> Let V=[Vx,Vy,Vz, 1]

[ARK SAID]
Why the fourth component of V must be 1? But
it is not important for our problem.

[RMK SAYS]
relative to t it seems to be a tradition.

> Then
> L(V)A = i(V)dA + d(i(V)A) = W + dU = Q
[ARK SAID]
In coordinates this is the same as my formula, because
i(V)dA has two terms, d(i(V)A) has two terms, altogether
four terms, but two of them cancel!

Do you want to see it?

[RMK SAYS] (grin)
I have better sources.

[ARK SAID]
Here it is:
Compute the first term:
1) (dA)_ij = d_i A_j - d_j A_i
[RMK would write]

$$A = A_x dx + A_y dy + A_z dz - \phi dt$$

$$dA = \{\partial A_k/\partial x^j - \partial A_j/\partial x^k\} dx^j \hat{} dx^k = F_{jk} dx^j \hat{} dx^k = \mathbf{B}_z dx \hat{} dy ... + \mathbf{E}_x dx \hat{} dt ...$$

where in usual engineering notation,

$$\mathbf{E} = -\partial \mathbf{A}/\partial t - grad\phi, \qquad \mathbf{B} = curl\ \mathbf{A}.$$

[ARK CONTINUES]

Therefore
2) [i(V)dA]_j = V^i (dA)_ij = V^i d_i A_j - V^i d_j A_i
Compute the second term (using Leibniz rule)

[RMK would write]

$$\begin{aligned} W &= i(V)dA = \mathbf{B}_z \mathbf{V}^x \hat{} dy - \mathbf{B}_z \mathbf{V}^y dx \ \ ... + \mathbf{E}_x \mathbf{V}^x \hat{} dt ... \\ &= (\mathbf{E} + \mathbf{V} \times \mathbf{B})_x dx + ... - (\mathbf{E} \cdot \mathbf{V}) dt \end{aligned}$$

[ARK CONTINUES]
3) d(i(V)A)_j = d_j [V^i A_i] = d_j (V^i) A_j + V^i d_jA_i

[RMK would write]

$$U = i(V)dA = A \cdot V - \phi$$

and

$$L(V)A = W + dU \tag{1}$$

The Lorentz force term "falls out" from the virtual work 1-form component W of the Lie derivative. **It does not involve derivatives of V.**

[ARK continues]
Now, when you add 2) and 3) , you see that second terms
cancel and we get simple
(L(V)A)_j = Vˆi d_i A_j + d_j (Vˆi) A_j

Which was my formula. Thus, as you see, Ark knows about
Lie derivative. Ark even wrote a paper (with Daniel Kastler)
"Graded Lie-Cartan pairs", and "Graded Lie-Cartan Pairs II"
Annals of Physics (NY) 179, No 2 (1987), 169-200
[RMK SAYS]

Wonderful, Now you know something more.
> For details of Cartan's magic formula see Marsden and Riatu
> and my website.
> **
[ARK SAID]
It is not necessary
> Recall that dA generates E and B as components of an 4 x 4 anti-symmetric
> matrix based on A. Sommerfeld's 6 vector. ** evaluate virtual work 1-form
> W = i(V)dA, get W= (E + VxB) dot dr - (E dot V) dt
> = Lorentz force times differential distance - (power) dt
> ****

[ARK SAID]
THis does not matter. Lie derivative of A with respect V
does contain partial derivatives of V while Lorentz term
does not have them.

[RMK SAYS]

But it is the virtual Work component, the non-exact component, of the Lie derivative which does **indeed** give the format of the Lorentz force.

> For currents of the type J = rho V

> Evaluate L(J) A = W +dU

> with virtual work evaluated as

> W = i(J)dA = (rho E + JxB) dot dr - (E dot J) dt

[ARK SAID]

THis does not matter. Lie derivative of A with respect V

does contain partial derivatives of V while Lorentz term

does not have them!!!!

AGREE??? Or should I be even more clear?

ark

[RMK says]

The important point is that the non-exact component of Lie derivative yields the Lorentz format precisely.

My original statement was:

By asking for how the 1-form of A and the 2-form F

propagate along a direction field

L(V) A = > Q,

the Lorentz Force law pops out automatically

as a derived quantity. IT is NOT added into the theory,

it falls out from the definition of the Lie derivative.

THIS DOES NOT SAY THE THE LORENTZ FORCE IS EQUAL TO THE LIE DERIVATIVE

The Lorentz force IS NOT equal to the Lie derivative acting on forms, unless the U term is an evolutionary constant. However, the format of the Lorentz force law comes form the virtual Work 1-form, W in all cases.

HOW CAN I BE MORE CLEAR?

regards

RMK