Changes made in the manuscript

1. Our main objective was to introduce for completely mixing systems a quantitative measure which in the classical case detects chaotic behavior of the system. In order to clarify this point the following definition has been added at the end of section 2:

Completely mixing systems in which $\lambda(f) > 0$ for some $f \in D$ will be called **exponentially mixing**. *As the example of r-adic transformation shows they are closely related to chaotic systems.*

2. In order to explain the appearance of the minus sign in formula (6) the following remark has been inserted to the text:

We have put the minus sign because our objective here is to replace characteristic exponents, which are defined in terms of trajectories, by a quantity expressed in terms of densities in such a way that they would measure the same property of the given dynamics, and hence coincide for simple one dimensional systems. As will be shown in proposition 2.2, in the r-adic case the Lyapunov characteristic exponent $\sigma(x_0) = \log r$, for any $x_0 \in (0, 1)$, indeed equals to our quantity $\lambda(\mathbf{1})$. Therefore, although this minus sign may seem to be artificial, it is necessary in order to describe the same feature of the dynamics.

3. A new example, suggested by the second Referee, of a two-level atom driven by a laser field has been presented in section 4.3. It turned out that for quantum systems the property of being exponentially mixing is weaker than in the classical case and so may not imply chaotic behavior. Therefore, the following comment has been added to the Introduction:

The property $\lambda_q > 0$ selects a subclass of completely mixing systems which we call exponentially mixing. However, contrary to the classical case, exponentially mixing quantum open systems may not imply chaotic behavior.

4. In order to explain why in the examples we used a simplified version of formula (13) the following remark has been added in section 3:

Finally, let us point out that if a completely mixing system has a stationary density matrix ρ_0, that is $T_t(\rho_0) = \rho_0$ for all $t \geq 0$, then ρ_0 is unique and λ_q does not depend on the choice of an initial statistical state ρ. In other *words* $\lambda_q(\rho) = \lambda_q(\rho_0)$, *where*

$$\lambda_q(\rho_0) = \inf_{\sigma \neq \rho_0} \lim_{t \to \infty} [-\frac{1}{t} \log \|T_t \sigma - \rho_0\|_1]$$

5. Conservative quantum systems, since they evolve in a unitary way, cannot be completely mixing, and so our strategy does not apply to them. A sentence about this point has been inserted to the text.

6. A number of other remarks which should make it easier to follow mathematical arguments have been also inserted.

7. Inspired by suggestions of the referees two new references [17] and [31] have been added, with comments relating their results to those of the present paper.

8. We have kept mathematical style of our presentation since we think that rigorous results enable a more efficient discussion. We hope that this will not discourage a potential reader.