On interaction between classical and quantum systems

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Abstract

We propose a mathematically consistent model of interaction between classical and quantum systems.

1 Introduction

According to Niels Bohr there is an imperative fundamental duality between the classical and the quantum levels of Nature. Our approach provides a mathematical form to such a view and thus transfers its contents from the realm of philosophy into that of physics. The model that we present below shows that a mathematically consistent description of interaction between classical and quantum systems is feasible. Following Niels Bohr we believe that the very fact that we can communicate our discoveries to our fellow-men constitutes an experimental proof that interactions of the type that our model describes do exist in Nature.

We believe that our approach straightens out some of the conceptual puzzles of quantum theory. From the point of view of quantum measurement theory our model has the following two attractive features: first, it shows that the predicted results of a would be quantum measurement on the initial quantum state can be read from the evolution of the classical system, and second, the initial state of the quantum system evolves into the mixture suggested by quantum measurement theory based on the projection postulate.

It is also quite possible that a similar model can provide e.g. a *selfconsistent* description of an interaction between quantized Fermi fields and classical gravitational

field, including the source action of the quantized matter on the classical geometry. In view of the known difficulties with the interpretation of quantized gravity and because of the growing interest in the black hole evaporation etc. further work along the lines that we draw in the present paper could put some new light on these interesting topics.
As a summary of our project we think that our results reduce the number of puzzles to one i.e. that of 'arrow of time 'whereas initially we believed that two important ones had to be solved i.e. the puzzle of irreversibility and that of quantum measurement. [1]. We also believe that this remaining puzzle can be solved only after we have acquired a radically new understanding of the nature of time. [2]
The most natural mathematical framework for our presentation would be $C^\star$ - algebraic one. However, in order to make our discussion as simple as possible we will use only matrix algebra. This has the advantage that the reader should easily be able to follow our mathematical reasoning. The price paid for this simplicity is that it will not be always evident why do we choose a particular form for the description of states, of observables and of the time evolution of the systems. There are two most essential ingredients in our model: *superselection rules* and *dissipative dynamics* . By superselection rules we mean here the classical parameters that label different Hilbert spaces that are needed for a complete description of a given system. Superpositions of pure states (i.e. vectors or, better, rays) from two Hilbert spaces with different labels are not observable. In our model the distinct pure states of the classical system (Dirac measures) define superselection rules of the total system. Thus we label coherent Hilbert spaces of the total system by the pure states of the classical system. Usually superselection quantities (e.g. electric charge) are taken to be constants of motion. It should be stressed that it is *not* the case with our model. Thus, to avoid misunderstandings, the very term 'superselection rules' should be avoided. A better name would be for instance 'central quantities', where the term 'central' means they are commeasurable with all other quantities, i.e. that the operators representing the quantities in question commute with all the observables of the system and thus belong to the *center* of its algebra of observables. [3] By *dissipation* we mean that the time evolution is generically described not by a unitary group but by a more general concept of a *completely positive semigroup*. The main characteristic of a dissipative evolution is that it does not map pure states into pure states. Thus it is well defined on the level of density matrices where it preserve convexity, positivity and trace, but not on the level of pure states i.e. vectors or rays. [4] Each of the ingredients, central quantities *and* dissipative evolutions have

[1] As exemplified, for instance, by the paradoxes of von Neuman's infinite chain, Schrödinger's cat and of Wigner's friend.

[2] An intensive research towards a similar end has been inspired by Ilya Prigogine (see e.g. Ref. [1] and, for a more technical account, Ref. [2] and the references quoted there).

[3] Thus, for example, time is a central quantity in Galilean-relativistic quantum mechanics (see Refs [3, 4, 5, 6]).

[4] Notice however that some *nonlinear and stochastic* evolutions of Hilbert space state vectors can lead to dissipative evolutions of statistical states. See e.g. Ref. [7] and references therein.

already been discussed separately before by other authors. It is only by putting them both into action simultaneously that we obtain a completely new perspective: central quantities representing the classical degrees of freedom can now evolve with time, with their evolution depending on the actual state of the quantum subsystem. It is thus by allowing for both central quantities and for dissipative dynamics that makes interaction possible between classical and quantum degrees of freedom.
The reader will notice that, according to the standard terminology, our joint system is *open*. Thus one is tempted to try to understand its behaviour as an effective evolution of a subsystem of a unitarily evolving larger, coherent, quantum system. Although mathematically possible, such an enlargement (the mathematical term is *dilation*) is, without further postulates, non-unique and neglects the fact that central quantities (like the electric charge) seem not only to exist but also to play a rather important role in our Universe. Therefore it is wise, in our opinion, to extend the prevailing paradigm and learn as much as possible how to deal directly with open systems and incomplete information.
It seems appropriate to try to answer a possible objection against our paper: one could object that once we have a positive semigroup then the problems of measurement as well as irreversibility are *automatically* resolved. But if so, if positive semigroups solve all the problems then, we may ask, why not to *assume* positive semigroups from the very beginning? The only non-metaphysical reason that we know is this: a lack of simple principles of universal validity. And it is the main objective of our paper to propose such a simple principle. We propose to discuss the hypothesis that the yes-no-flip mechanism that we exploit in our model may constitute an *elementary building block* used by Nature in the communication between Her quantum and classical levels. [5]
The philosophical boost for this investigation came from the works of Niels Bohr and Karl Popper. The physical ideas were influenced by the works of E.T. Jaynes and also of Ilya Prigogine and his school. The mathematical model came out mainly from our studying of the papers of Josef Maria Jauch [8, 9], Klaus Hepp [10], Constantin Piron [3, 4, 5] , Nicolas Gisin [11, 12, 7] and Huzichiro Araki [13] , and also from a few sentences in the book by Hans Primas [14].

2 The model

In our model we will consider a quantum system q in an interaction with a classical system c. We assume that the pure states of q are given by rays in a complex Hilbert space H_q. For the observable algebra of q we take the algebra $A_q = L(H_q)$ of all bounded linear operators on H_q. The statistical states of q are then given by positive operators on H_q of trace 1. We denote by S_q the convex set of all these states. Our classical system will need to have only finite number of distinct pure

[5] We thank I.E. Antoniou for his criticism. It prompted us to express more explicitely our actual position.

states. If X_c denotes the set of pure states of c, then A_c is the Abelian algebra of complex functions on X_c [6], while S_c is the space of probability measures on X_c. We assume that there is a distinguished, finite, family of mutually orthogonal nonzero projections $e_1, ..., e_n$ in H_q. Such a family may come from spectral resolution of a distinguished observable of q - for instance of an observable to be measured. In that case we would have $e_1 + \ldots + e_n = I$. In order to allow our formulae to be slightly more general than necessary, we will however not assume that the projections e_i add to I. Thus we will put $e \doteq \sum_{r=1}^{n} e_r$ and $f \doteq I - e$. Notice that a measurement of $e_1, \ldots, e_n$ is automatically a measurement of f. The discussion below will cover both cases, $f = 0$, and $f > 0$. In any case our classical system c is supposed to have at least $n + 1$ distinct pure states. We will take the convention that greek indices run through $0, \ldots, n$, while the latin ones through $1, \ldots, n$. For the set of classical states we shall use the notation $X_c = \{s_\alpha\} = \{s_0, ..., s_n\}$. The state s_0 will be a distinguished state and will play the role of the initial (or 'neutral') state. Then A_c - the algebra of complex functions on X_c - is isomorphic to $\mathbb{C}^{n+1}$. States $p \in S_c$ of c are $(n + 1)$-tuples $P = (p_0, ..., p_n)$, $p_\alpha \geq 0$, $\sum_\alpha p_\alpha = 1$. The joint, total system, $tot \doteq q + c$ has as its algebra $A_{tot} \doteq A_q \otimes A_c = L(H_q) \otimes \mathbb{C}^{n+1}$. It is convenient to describe A_{tot} as an algebra of operators on $H_{tot} \doteq H_q \otimes \mathbb{C}^{n+1}$. The element of A_{tot} are then block diagonal $(n+1) \times (n+1)$ matrices $A = diag(a_0, ..., a_n)$, whose entries a_α are bounded linear operators on H_q. The quantum algebra is embedded into A_{tot} via the embedding i_q:

$$i_q : a \in L(H_q) \mapsto a \otimes I = diag_{n+1}(a, ..., a). \tag{1}$$

The classical algebra A_c is embedded into A_{tot} via the map i_c

$$i_c : \lambda = (\lambda_0, ..., \lambda_n) \mapsto diag(\lambda_0 I, ..., \lambda_n I),\ \lambda_\alpha \in \mathbb{C}. \tag{2}$$

States $W \in S_{tot}$ of tot are given by block-diagonal matrices

$$W = diag(w_0, ..., w_n), \tag{3}$$

where $w_\alpha \in L(H_q)$, $w_\alpha \geq 0$, $\sum_\alpha Tr(w_\alpha) = 1$. For the expectation value of an observable $A \in A_{tot}$ in a state $W \in S_{tot}$ we have $W(A) = \sum_\alpha Tr(w_\alpha a_\alpha)$. Each state W of A_{tot} projects, by taking partial traces, onto the states $\pi_q(W)$ of A_q and $\pi_c(W)$ of A_c respectively. We have

$$\pi_q(W) = \sum\nolimits_\alpha w_\alpha, \tag{4}$$

and

$$\pi_c(W) = (Tr(w_0), ..., Tr(w_n)). \tag{5}$$

Thus

$$Tr(W\ i_q(a)) = Tr(\pi_q(W)\ a)$$

[6] Notice that because X_c is finite here, we do not have to worry about continuity of these functions. In particular the Dirac measures are continuous.

and

$$Tr(W\ i_c(\lambda)) = \sum\nolimits_\alpha \pi_c(W)_\alpha \lambda_\alpha.$$

Given states $P = (p_0, ..., p_n) \in S_c$ and $w \in S_q$, their product

$$w \otimes P \doteq diag(p_0 w, ..., p_n w) \tag{6}$$

projects via the maps π_c and π_q on P and w respectively. It corresponds to a unique state of the joint system $tot = q + c$ that can be described by the words: 'q is in the state w , c is in the state P , and there are no correlations between both'.

We consider now time evolution of the joint system. We assume that it is given by a semigroup α^s , $s \geq 0$, of completely positive maps [7] of A_{tot} , with $\alpha^s(I) = I$. We recall the reader that an element b of a $C^\star$-algebra B is *positive* iff it is of the form $b = a^\star a$ for some $a \in B$. A linear map $\alpha : B \to B$ of B into itself is said to be *positive* if it maps positive elements of B into positive ones; it is called *m-positive* if $\alpha \otimes I_m$ is a positive map of $B \otimes M_m$, M_m being the $m \times m$ complex matrix algebra. Finally α is said to be *completely positive* if it is m-positive for all m. [8] Time evolution of states is then given by the dual maps $\alpha_s : S_{tot} \longrightarrow S_{tot}$ with

$$\alpha_s(W)(A) = W(\alpha^s(A)).$$

It follows directly from the definition that α_s maps states into states, preserving their positivity and normalization (but *not* necessarily pure states into pure states). In the following we shall not use any of these $C^\star$-algebraic concepts. Instead, we will use the fact that owing to the theorems by Stinespring and Lindblad (cf. Refs. [16, 17]) α^s must be be of the form:

$$\alpha^s = exp(sL), \tag{7}$$

with L of the form: [9]

$$L(A) = [iH, A] + \sum\nolimits_{\iota=1}^{N} V_\iota A V_\iota^\star - \frac{1}{2}\{R, A\}, \tag{8}$$

where

$$R \doteq \sum\nolimits_\iota V_\iota V_\iota^\star \in A_{tot}, \tag{9}$$

V_ι being arbitrary linear operators in H_{tot} such that

$$\sum\nolimits_\iota V_\iota A V_\iota^\star \in A_{tot}\ whenever\ A \in A_{tot},$$

while H is an arbitrary Hermitian operator from A_{tot}:

$$H = H^\star \in A_{tot}.$$

[7] We adhere here to the view that the *positive* and *convex* structure is more primary than the *algebraic* one (cf. e.g. Ref. [15]).

[8] A brief discussion of the relevant concepts can be found in Ref. [18], cf. also Ref. [19] .

[9] Notice that the representation of L in terms of V_r is not a unique one.

Notice that we have $L(I) = 0$, and so $\alpha^s(I) \equiv I$ for all $s \geq 0$. Let us define $W(s) = \alpha_s(W)$. The above evolution of observables of the system leads to the following Liouville evolution equation for states:

$$\dot{W}(t) = -i[H, W(t)] + \sum\nolimits_\iota V_\iota^\star W(t) V_\iota - \frac{1}{2}\{W(t), R\}. \tag{10}$$

The condition $L(I) = 0$ translates here to $Tr(\dot{W}) = 0$ or, equivalently, $Tr(W) \equiv 1$ automatically guaranteed by the above form of the Liouville equation.[10] In our model we will neglect completely the Hamiltonian part of the interaction and concentrate on the dissipative part. The reason for this being that it is only the dissipative part that matters for interactions between classical and quantum subsystems. Indeed, the observables A_c of the classical subsystem form the *center* of the total algebra $A_{tot} = A_q \otimes A_c$, and the center of any algebra is invariant under all automorphisms; in particular it is invariant under any automorphic unitary evolution. In other words the Hamiltonian part of the Liouville operator L is always passive in the process of transfer of information from a quantum subsystem to a classical one. [11] And it is only the dissipative part that can cause such a transfer *in a finite time* .

We propose a simple Liouville operator that describes an interaction of q and c: it is of the form (8) with $N = n$, $r = 1, 2, ..., n$ and $V_r = e_r \otimes \sigma_r$, where σ_r is the *flip* transformation of X_c that transposes s_0 with s_r. [12] We can write V_r explicitly as a block matrix [13] in H_{tot}:

$$V_1 = \begin{pmatrix} 0, & e_1, & 0, & \dots, & 0, & 0 \\ e_1, & 0, & 0, & \dots, & 0, & 0 \\ 0, & 0, & 0, & \dots, & 0, & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0, & 0, & 0, & \dots, & 0, & 0 \end{pmatrix},$$

[10] It is to be noticed that the quoted theorem of Lindblad (cf. also the result by Gorini and Kossakowski, Ref. [20]) on generators of completely positive semigroups assumes *norm-continuity*. This assumption presents no problems in the finite-dimensional case that we are discussing. It needs not be however satisfied in a model with H_q infinite dimensional or X_c infinite.

[11] Notice however that additional effects may result if the conservative and dissipative parts of the evolution do not commute.

[12] One can arrive at this form by applying the theorem of Arveson on *extremal* completely positive maps (cf. Ref [21]) to the Christensen form (see e.g. Ref. [19], p.240, and refrences there) of a generator of a completely positive semigroup. One arrives in this way at *simple* generators. They form a basis in the convex space of all generators of CP semigroups. Our generator is not a simple one, but a symmetric combination of two such (an antisymmetric one acts exactly the same way). We could put any operators in place of the e_r-s, but not the unitaries as they would not give rise to a nontrivial interaction of our two subsystems.

[13] The idea behind this particular form is that it corresponds to a process in which the logical values *yes* or *no* of the observables e_r are being recorded. This would give a maximal possible information gain about the quantum subsystem iff the projections e_r are minimal and add to I (i.e. $f = 0$). One could think, for instance, that each 'elementary e_r *yes-no* event' induces a 'flip' $s_0 \leftrightarrow s_r$ in the classical subsystem.

$$V_2 = \begin{pmatrix} 0, & 0, & e_2, & \dots, & 0, & 0 \\ 0, & 0, & 0, & \dots, & 0, & 0 \\ e_2, & 0, & 0, & \dots, & 0, & 0 \\ 0, & 0, & 0, & \dots, & 0, & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0, & 0, & 0, & \dots, & 0, & 0 \end{pmatrix},$$

$$\dots$$

$$V_n = \begin{pmatrix} 0, & 0, & 0, & \dots, & 0, & e_n \\ 0, & 0, & 0, & \dots, & 0, & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0, & 0, & 0, & \dots, & 0, & 0 \\ e_n, & 0, & 0, & \dots, & 0, & 0 \end{pmatrix},$$

The Liouville evolution equation (10) takes now the following explicit form:

$$\dot{w_0} = \sum_{r=1}^{n} (e_r w_r e_r - \frac{1}{2}\{e_r, w_0\}), \tag{11}$$

$$\dot{w_r} = e_r w_0 e_r - \frac{1}{2}\{e_r, w_r\}. \tag{12}$$

These equations can be solved explicitly by summing up of the Taylor series. A straightforward computation gives:

$$\begin{aligned} W(t) = \ & \tfrac{1}{2}(V(W) + E(W)) + EWE - \{E, W\} + W + \\ & + e^{-\frac{1}{2}t} (\{E, W\} - 2EWE) + \\ & + e^{-t} (EWE - E(W)) + \\ & + \tfrac{1}{2} e^{-2t} (E(W) - V(W)), \end{aligned} \tag{13}$$

where we have put W for $W(0)$ and used the notation

$$\begin{aligned} E_r &\doteq V_r^2, \\ E &\doteq \sum_{r=1}^{n} E_r, \\ V(W) &\doteq \sum_{r=1}^{n} V_r W V_r, \\ E(W) &\doteq \sum_{r=1}^{n} E_r W E_r. \end{aligned} \tag{14}$$

We shall apply these formulae to the particular initial state $W(0) = w \otimes P$ which is a product of a state w of the quantum subsystem and of a state $P(0) \doteq (p_0, p_1, \ldots, p_n)$ of the classical one. Then for $W(t) = diag(w_0(t), \ldots, w_n(t))$ we get: [14]

$$w_0(t) = p_0(\frac{1}{2}e(w) + fwf) + \frac{1}{2}\sum\nolimits_r p_r e_r w e_r + \tag{15}$$

$$+e^{-\frac{1}{2}t}p_0(ewf + fwe) +$$

$$+e^{-t}p_0(ewe - e(w)) +$$

$$+\frac{1}{2}e^{-2t}(p_0 e(w) - \sum\nolimits_r p_r e_r w e_r),$$

$$w_i(t) = \frac{p_0}{2}e_i w e_i + p_i(w + \frac{3}{2}e_i w e_i - \{e_i, w\}) + \tag{16}$$

$$+e^{-\frac{1}{2}t}p_i(\{e_i, w\} - 2e_i w e_i) +$$

$$+\frac{1}{2}e^{-2t}(p_i e_i w e_i - p_0 e_i w e_i),\ i = 1, \ldots n\ ,$$

where we have used the notation: $e(w) \doteq \sum_r e_r w e_r$, $e \doteq \sum_r e_r$, $f \doteq 1 - e$.

In the following we shall specialize to the case of $f = 0$, i.e. $e = e_1 + \ldots e_r = I$. It is instructive to consider a special class of initial states $P^\epsilon(0)$ of the classical system parametrized by ϵ, $0 \leq \epsilon \leq 1$:

$$p_0(0) = 1 - \frac{n\epsilon}{n+1}, \tag{17}$$

$$p_i(0) = \frac{\epsilon}{n+1}. \tag{18}$$

Thus for $\epsilon = 0$ the classical system starts from a pure state $P(0) = (1, 0, \ldots, 0)$, while on the other extreme, for $\epsilon = 1$, it starts from the maximal entropy state $P(0) = (\frac{1}{n+1}, \ldots, \frac{1}{n+1})$. By taking trace of in equation (16) we can compute now $p_i(t) \doteq Tr(w_i(t))$, and thus the normalized distribution

$$\tilde{p}_i(t) \doteq \frac{p_i(t)}{\sum_r p_r(t)} \tag{19}$$

as read off from the outputs $s_1, \ldots, s_n$ of the classical system. The result reads:

$$\tilde{p}_i(t) = q_i + \frac{\epsilon(1 - nq_i)}{\epsilon n + \frac{(1-\epsilon)(n+1)}{2}(1 - e^{-2t})}, \tag{20}$$

where we introduced the notation

$$q_i \doteq Tr(e_i w), \tag{21}$$

[14] We have found **REDUCE** quite useful in some of these computations.

for the initial quantum probabilities to be measured. We see by inspection that for $\epsilon = 0$ we have

$$\tilde{p}_i(t) \equiv q_i \tag{22}$$

for all $t > 0$. That means that in this case the measurement gives a result that coincides with the measured distribution $\{q_i\}$ *immediately* after switching on of the interaction. In the other extreme, for $\epsilon = 1$, we get instead

$$\tilde{p}_i(t) \equiv \frac{1}{n} \tag{23}$$

for all $t \geq 0$. The maximum entropy state is a *stationary* state of the classical subsystem and, in this case, we get no information at all about the quantum system by observing the time evolution of the classical one. In the intermediate regime, for $0 < \epsilon < 1$, we deduce from the formula (20) that $|\tilde{p}_i(t) - q_i)|$ decreases at least as $2\epsilon(1 + e^{-2t})$ with $\epsilon \to 0$ and with $t \to \infty$.
For $\epsilon = 0$, that is when the measurement is exact, we get for the limiting partial state of the quantum subsystem (by using the formula (4) applied to (15,16)):

$$w_q(t) = e(w) + e^{-t}(w - e(w)), \tag{24}$$

so that

$$w_q(\infty) = e(w) = \sum\nolimits_r e_r w e_r. \tag{25}$$

Thus, in this case, not only the quantum probabilities q_i are exactly mirrored by the state of the classical system, but also the partial state of the quantum subsystem tends to a limit which agrees with that required by the standard quantum measurement projection postulate. Notice that the relative distance between $w_q(t)$ and its $t = \infty$ limit is given by the formula:

$$\frac{d(w_q(t), w(\infty))}{d(w_q(0), w(\infty))} = 1 - e^{-t} \tag{26}$$

where the distance $d(w_1, w_2)$ is given by $Tr(|w_1 - w_2|)$

An important comment is due at this place. The particular model described above is minimal with respect to the number of classical degrees of freedom involved. It is mathematically the simplest one but physically not realistic. There are two immediate ameliorations of the model which are based on the same principle and have exactly the same form of time evolution (13). The first improvement consists of replacing one 'reservoir' s_0 with n reservoirs $s_{01}, \ldots s_{0n}$, and taking $p_{0r} = 1/n$, $p_r = 0$ for the initial classical state. This version is better suited to meet the locality demands, but it is statistically inefficient (for each event there is only $1/n$ chance of registering a 'flip '). The second improvement consists of putting an independent classical flipping device for each of the n quantum yes-no alternatives. Now the classical system has 2^n states parametrized by sequences $\{s_1, \ldots s_n\}$, $s_r = 0, 1$ and the initial classical (pure) state is the sequence $\{0, \ldots, 0\}$. The operators V_r are

products of e_r with the flips on r-th place. The conclusions are exactly as before as they are based on the same *yes - no - flip* algebra that leads to the semigroup (13). The probability that the r-th device s_r will flip from the initial value 0 to 1 during time t is

$$p_{r,0\to 1}(t) = \frac{1}{2}(1 - 2e^{-2t})q_r. \tag{27}$$

3 Concluding Remarks

We formulated a model that provides an answer to one of the important conceptual problems of quantum theory, the problem of *how and when a quantum phenomenon becomes real* as a result of a suitable dissipative time evolution. In our model we did not attempt to indicate any particular physical mechanism that leads to the kind of interaction that we have discussed. We leave this question open for future investigations. As we have remarked in the introduction there may be more than one answer to this question (as for instance hidden variables and/or stochastic realizations). At the present stage we would prefer to refrain from stating additional hypotheses. However, we want to stress one point: when there are several competing individual descriptions that lead to our Liouville equation for statistical states, then the priority should be given to those that better tackle the problem of locality. The last mutation of our model, with n independent flipping devices, described at the end of the previous section seems to be best suited in this respect.

As we have already noticed, with a properly chosen initial state, our model gives an immediate exact mirroring of the quantum probability distribution in the statistics of flips of the classical system. Although mathematically correct, this statement should be properly understood. First of all each real implementation of the model will have a characteristic time-scale τ, and time t will appear in the exponentials as t/τ. As seen from the Eq. (27) one has then to wait $(ln2/2)\tau$ for one of the devices $s_1, \ldots, s_r$ to flip with probability 0.25. During this time the quantum state travels $1 - 1/\sqrt{2} \approx 0.3$ of its distance (cf. Eq. 26)) towards $e(w)$. To reduce this distance is to increase the number of necessary data by about the same factor. A nondemolition measurement, i.e. such that the quantum state does not change much during the interaction interval , would then need proportionally more of data to get a reasonable statistics.

The central idea of these models is based on a modification of quantum mechanics by introducing dissipative elements in the basic dynamical equation and on allowing for a nontrivial dynamics of central quantities. In conclusion it is tempting to interpret physically the relaxation time τ. In fact modern technological developments allow a complete measurement in very short time of the order of nanoseconds. On the other side in applications to quantum cosmology τ can be of the order of age of the Universe. A few words may be also useful to suggest possible application of models of this type in biology. Living organism are coherent open systems with a program dependent of molecular recording processes. Variations take place

there on the quantum level and are translated and amplified to generate macroscopic variations.

Last but not least there comes the question about a continuous limit of the model (as for instance in a measurement of the position). There are four evident options here: naive coarse-graining, using nonstandard analysis (cf. Ref. [22]), using non-normal expectations (thus states are no longer described by density matrices - cf. Ref. [23]) or using gaussian filters and means over groups (cf. Ref. [24]). The last option seems to be the easiest to implement although the idempotent property is lost and the simple semigroup formula (13) becomes replaced by an infinite series in $exp(-kt)$. Even if a continuous implementation of the flip mechanism may do approximately the same job as the discrete one, it must be remarked that the natural 'milieu' for this mechanism is a discrete one. [15]

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[15] Thus if the universality hypothesis expressed in the Introduction is to be taken seriously, it will naturally lead to consequences that are at variance with some of the elements of the prevailing paradigm. Space (and thus also time) is to be build out of the discrete elements.

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