VANISHING VIERBEIN IN GAUGE THEORIES OF GRAVITATION [1]

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Abstract

We discuss the problem of a degenerate vierbein in the framework of gauge theories of gravitation. We show that a region of space-time with vanishing vierbein but smooth principal connection can be, in principle, detected by scattering experiments.

Author's comments:
This paper was send for publication in February 1984. It then took almost a year to get Referee Report. Here its is:

> This paper contains a summary of some of the known aspects of gravity as a gauge theory and addresses, without substantive results, the phenomena associated with regions where the vierbein vanishes. What is new in the paper is connected with this latter question, but I find the discussion misleading and in any case not sufficiently well developed to justify publication in the paper's present form. Coordinate regions where the metric or vierbein vanishes must be treated with considerable care, as there are generally identifications of points to be taken into account. Pictures

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such as that of Fig. 2 are thus very misleading. The vanishing of the vierbein is not in general a coordinate-independent statement and what seems to be two points in one coordinate frame could well be seen to be just one in another. This is the case, e.g., with a uniformly accelerated frame in the x direction, where all points with $x = 0$ and τ finite are to be identified.

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My comments on the above report - as of today September 18, 1999:

In retrospect, I realize that the anonymous referee did not, in fact, reject the paper. He suggested that it did not justify publication in its present form. However, I was sufficiently discouraged by his comments to decide not to work on improving the form any further. Also in retrospect, it seems clear that the referee missed my point: when all the information that we have is included in the metric then, we can try to play the game of identifying the points to get rid of the "singularity." But within the framework I was discussing in this paper, there was also a principal connection using extra dimensions and giving extra-information. Gluing some space-time points together would create discontinuity of the connection. Vierbein, in the paper, was defined as one-form with values in an associated vector bundle and its vanishing was a coordinate independent statement. Yet, the referee was right on one point: the ideas of the paper could have been developed better. Although I have shied away from the subject in the intervening years, at the time I was suggesting that "faster-than-light teleportation" is possible but, indeed, I failed to provide the explicit description of a transdimensional remolecularizer working on this principle.

In the meantime the paper has been quoted in I. Bengtsson, "Degenerate metrics and an empty black hole", *Class. Quantum*

Grav. **8** 1991, 1847-1857 (for more recent developments cf. also I. Bengtsson and T. Jacobson, "Degenerate metric phase boundaries", *Class. Quantum Grav.* **14** 1997, 3109-3121

1 Introduction

Hanson and Regge [1] (see also D'Auria and Regge [2]) suggested that a gravitational Meissner effect might exist producing torsion vortices accompanied by vanishing of a vierbein. This in turn may indicate a phenomenon of "unglueing" of a principal 0(4) (or 0(3, 1)) bundle from the bundle of frames of the space-time manifold M. The idea that a proper arena for gauge theories of gravitation is an external principal bundle P over M, rather than the frame bundle LM of M, has been put forward by many authors (see e.g. [3, 4, 5]). It is not our aim to justify this belief here. In fact what is relevant is not so much the choice of a bundle but the choice of dynamical variables and their dynamics. As long as a vierbein is thought of as representing a homomorphism of the two bundles, any distinction between P and a subbundle of LM does not really matter. The distinction becomes important when the vierbein is interpreted as a linear map $\theta : \zeta^\mu \mapsto \theta^a_\mu \zeta^\mu$ from the tangent spaces of M into the fibers of a vector bundle associated with P. Such an interpretation is natural in gauge theories of the Poincaré group $IO(3,1)$ (see e.g. [4, 5, 6]) and of $O(3,2)$ [8], where θ appears as a composite object.

The paper is organized as follows. in Sect. 3 we formulate a gauge theory of the Lorentz group and point out that the requirement of smoothness of the Lagrangian at a degenerate vierbein is a strong selection criterion. Only three terms survive the test, one of them having a "wrong parity", and two others are (cosmetically improved) the standard Einstein-Hilbert Lagrangian and the cosmological term. In Sect. 4 we briefly discuss the structure of the gauge theory of the Poincare group, and of the $O(3,2)$ gauge theory considered by Stelle and West [8]. In Sect. 5 we analyse relations between these abstract gauge theories and the conventional ones based on metric tensor and an affine connection. The relation can be visualized as follows. In an open region where the vierbein θ is nondegenerate the external bundle is glued with the help of θ to the frame bundle and the bundle connection can be interpreted as an affine connection on M. When θ becomes degenerate the external bundle detaches from the frame bundle. The principal connection chooses to live in the external bundle and the affine connection dies. Two examples are given in Sect. 6 . The first is a kind of gravitational instanton considered by Hanson and Regge [1] and D'Auria and Regge [2]. A zero of a vierbein is accompanied by non-zero torsion. The second example is an adaptation of a model discussed by Einstein and Rosen [9]. The vierbein vanishes here on a 3-dimensional "bridge" connecting two mirror copies of the exterior Schwarzschild universe. It is interesting to notice that the principal connection continues smoothly over the bridge and need not be regularized as in the first example. The Einstein–Rosen bridge is therefore a torsion-free regular solution of vacuum field equations of $0(3,1)$ gauge theory.

It was pointed out in [2] that "the vanishing vierbein at some point is not a disastrous feature of theory". It is one of the aims of the present note to point out that with an appropriate dynamics vanishing vierbein in a whole region need not be a disaster either. It is shown in Sect. 7 that such a "dead" region can have observable effects seen from outside, and that it introdu ces statistical elements (that is "freedom of choice") already on a classical level.

2 Notation

Let (P, π, M, G) be a principal bundle, let ρ be a representation of G on a vector space F, and let ρ' denote the derived representation of the Lie algebra $Lie(G)$ of G. For every $h \in Lie(G)$ let $\tilde{h}$ denote the fundamental vector field on P generated by h. An F-valued p-form, $p \leq dim(M)$, ϕ on P is called tensorial of type ρ if $i(\tilde{h})\phi = 0$ and $\mathcal{L}_{\tilde{h}}\phi = -\rho'(h)$ for all $h \in Lie(G)$ (see [10, p. 75]). Let $Q(\rho) = P \times_G F$ be the vector bundle associated with P via the representation ρ ([10, p. 55]). One can identify F-valued tensorial p-forms of type ρ on P with $Q(\rho)$-valued p-forms on M (see [10, p. 76], [12, Ch. VII. 1], [11, Ch XX. 5]). This identification is extensively used in the literature and we shall use it too without further references. In particular the curvature 2-form $\Omega = D\omega$ of a principal connection ω on P can be thought of as a 2-form on M with values in $Q(Ad)$, where Ad denotes the adjoint representation of G on $Lie(G)$.

3 $O(3,1)$ gauge theory

Let $(P, \pi, M, O(3,1))$ be a principal bundle over a 4-dimensional base manifold M. The fibers of P are to be thought of as being a priori completely detached from fibers of the frame bundle of M. Let ρ_0 denote the natural representation of $O(3,1)$ on $\mathbf{R}^4$. The dynamical variables of a generalized Einstein–Cartan theory are: a principal connection ω on P, and a $Q(\rho_0)$-valued 1-form θ on M. In a "pure gauge theory" [3] a Lagrangian 4-form should be constructed out of ω and θ alone. We demand that the action should be a smooth (and thus nonsingular) function of field configuration variables (ω, θ). In particular $\mathcal{L}$ should be continuous at $\theta = 0$. Among geometrical objects at our disposal we find only six candidates which have this property:

$$\mathcal{L}_1 = \epsilon_{abcd}\, \theta^a \wedge \theta^b \wedge \theta^c \wedge \theta^d \quad (1)$$

$$\mathcal{L}_2 = \epsilon_{abcd}\, \theta^a \wedge \theta^b \wedge \Omega^{cd} \quad (2)$$

$$\mathcal{L}_3 = \theta^a \wedge \theta^b \wedge \Omega_{ab} \quad (3)$$

$$\mathcal{L}_4 = D\theta^a \wedge D\theta_a \quad (4)$$

$$\mathcal{L}_5 = \epsilon_{abcd}\, \Omega^{ab} \wedge \Omega^{cd} \quad (5)$$

$$\mathcal{L}_6 = \Omega_{ab} \wedge \Omega^{ab} \quad (6)$$

Variations of $\mathcal{L}_6$ and $\mathcal{L}_5$ are exact 4-forms and do not contribute to classical field equations. Owing to the first Bianchi identity $\mathcal{L}_4$ differs from $\mathcal{L}_3$

[3] By a pure gauge theory we mean a gauge field theory where the primitive fields have no direct connection to space-time geometry. Such a theory should "interprete itself" (see Ref. [13].

by an exact form only. $\mathcal{L}_3$ has internal parity different from that of $\mathcal{L}_2$ and $\mathcal{L}_1$, and should not be combined with them unless P is reduced to $SO(3,1)$, which would imply dynamically preferred orientation of M - a viable possibility.[4] Introducing arbitrary coupling constants $\lambda_1, \lambda_2, \lambda_3$, Euler-Lagrange equations for

$$\mathcal{L} = (\lambda_1/4!)\mathcal{L}_1 + (\lambda_2/2)\mathcal{L}_2 + (\lambda_3/2)\mathcal{L}_3 \tag{7}$$

are:

$$\epsilon_{abcd}\theta^b \wedge ((\lambda_0/3!)\theta^c \wedge \theta^d + \lambda_1\Omega^{cd}) + \lambda_2\theta_b \wedge \Omega^{ab} = t_a, \tag{8}$$

$$\lambda_1\epsilon_{abcd}D\theta^c \wedge \theta^d + (\lambda_2/2)(D\theta_a \wedge \theta_b - D\theta_b \wedge \theta_a) = s_{ab}, \tag{9}$$

where t_a and s_{ab} are 3-forms representing the sources: energy-momentum and spin. It is important to notice that field equations (8) and (9) make sense for all smooth configurations (ω, θ), in particular for those with degenerate θ-s. However, the predictive power of these equations falls down with the rank of θ.

4 Gauge theories of $IO(3,1)$ and $O(3,2)$

Replacing $O(3,1)$ with $IO(3,1)$ one gets a gauge theory of the Poincaré group. The two important representations of $IO(3,1)$ on $\mathbf{R}^4$ are $\rho_0(a,\Lambda) : x \mapsto \Lambda x$, and $\rho(a,\Lambda) : x \mapsto \Lambda x + a$. The dynamical variables of an $IO(3,1)$ gauge theory are (cf. [4, 5]): a principal connection ω on $(P, \pi, M, IO(3,1))$, and a $Q(\rho)$-valued 0-form ϕ on M, $Q(\rho)$ being the affine bundle over M associated to ρ. The admissible Lagrangians are those of the $0(3,1)$ gauge theory but now θ is not a primitive field – it is defined by $\theta = D\Phi$. Given a field configuration (ω, Φ) one can always adapt a gauge (that is to use the translational freedom and choose a cross section of the principal $IO(3,1)$

[4]Another possibility is to introduce an extra pseudo-scalar field ψ and to replace $\mathcal{L}_3$ with $\psi\mathcal{L}_3$ (see [14]).

bundle) in such a way that $\Phi^a \equiv 0$ – thus, effectively, eliminating the field Φ from the theory. In this gauge the soldering form θ , defined above as $\theta = D\Phi$, coincides with the translational part ω^a of the connection ω. This closes our discussion of the Poincaré group gauge theory: after gauge fixing it effectively reduces to Lorentz group gauge theory.

Another viable possibility is a gauge theory of $0(3,2)$, with five dimensional fibres, as discussed in [8]. The dynamical variables of this theory are: an $0(3,2)$ – connection ω and a $Q(\rho)$ -valued 0-form y, where ρ denotes the natural representation of $O(3,2)$ on $\mathbf{R}^5$. The Lagrangian 4-form is [8]

$$\mathcal{L} = \epsilon_{ABCDE}\; y^A \wedge \Omega^{BC} \wedge \Omega^{DE},\; A, B, \ldots, E = 0, 1, 2, 3, 5 \tag{10}$$

together with a constraint

$$y^A y_A = const. \tag{11}$$

Given a configuration (ω, y) one can always adapt gauge in such a way that $y^5 = 0$. The (generalized) vierbein θ can be defined by imposing the condition

$$\theta^a = (Dy)^a,\; a = 0, 1, 2, 3 \tag{12}$$

in the adapted gauge.

5 Relation to metric-affine theories

Let (ω, θ) be a field configuration of an $O(3,1)$ gauge theory as discussed in Sect. 3. A point $p \in M$ is said to be a critical point of θ if $\det(\theta^a_\mu(p)) = 0$, otherwise p is called a regular point. The set M_θ of all regular points of a smooth θ is an open subset of M. When $M_\theta \neq M$ then θ is called degenerate. A linear space-time connection ∇ in TM (sometimes also called an affine connection on M; notice that we admit torsion here) is said to be *compatible* with a given field configuration (ω, θ) if

$$\theta(\nabla_X Y) = i(X)Di(Y)\theta \tag{13}$$

i. e. if θ is parallel with respect to (ω, ∇) when considered as a cross section of the fiber product $Q(\rho_0) \times T^\star M$. We have then

$$\theta(\nabla_X Y - \nabla_Y X - [X,Y]) = (D\theta)(X,Y) \tag{14}$$

and so the $Q(\rho_0)$ – valued 2-form $\Theta = D\theta$ may be considered as a generalized torsion.

Given a configuration (ω, θ) there exists a unique ∇ on M_θ, M_θ being the open submanifold of M consisting of all regular points of the vierbein, compatible with (ω, θ). Outside of M_θ – on the set of critical space–time points – the affine connection ∇ will not, in general, exist. To see this observe that at regular point we have, as a consequence of Eq. (13):

$$\Gamma^\sigma_{\mu\rho} = {\theta^{-1}}_a{}^\sigma(\partial_\mu \theta^a_\nu + {\omega_\mu}^a{}_b \theta^b_\nu) \tag{15}$$

where $\Gamma^\sigma_{\mu\nu}$ are the coefficients of ∇ in a coordinate system x^μ. From Eq. (15) we get

$$\Gamma_\mu \equiv \Gamma^\sigma_{\mu\sigma} = \partial_\mu \ln|\det\theta| \tag{16}$$

so that the part of ∇ which is responsible for the parallel transport of the length scale depends on the vierbein only (and not on the connection ω). When θ becomes degenerate then $\ln(\det\theta)$, and therefore also Γ_μ, diverge. If θ is identically zero then the argument does not apply, and inside such a region any affine connection is compatible with such a configuration.

For every configuration (ω, θ) one defines a covariant "metric" tensor $g_{\mu\nu} = \eta_{ab}\theta^a_\mu\theta^b_\nu$, where $\eta = diag(-1,+1,+1,+1)$ is the diagonal constant matrix. The induced scalar product in T_pM is nondegenerate if and only if p is regular. On M_θ we have then $\nabla_\mu g_{\nu\sigma} = 0$, where ∇ is a unique affine connection compatible with (ω, θ). The standard Einstein–Hilbert Lagrangian density $\mathcal{L}_{EH}$ of (g, θ) is

$$\mathcal{L}_{EH} = (\lambda_1 + \lambda_2 R)|\det(g)|^{1/2} \tag{17}$$

where R is the scalar curvature of (∇, g). One easily finds that

$$\mathcal{L}_{EH} = sgn(\det\theta)((\lambda_1/4!)\mathcal{L}_1 + (\lambda_2/2!)\mathcal{L}_2)_{0123} \tag{18}$$

where Φ_{0123} denotes the $(0,1,2,3)$ – component of a 4-form Φ.

The identity (18) holds on M_θ . Outside of this region the left hand side of Eq. (18) is not defined. The right hand side is "almost" defined - if not for the $sgn(\det\theta)$. It is thus clear why taking (7) for the Lagrangian four–form is a better choice.

6 Two examples of configurations with degenerate vierbein

Example 1.: Hanson and Regge [1] (see also [2]) consider an $O(4)$– thus Euclidean – version of Einstein-Cartan gauge theory. The base manifold M is $\mathbf{R}^4$ and the vierbein (θ^a_μ) is defined by

$$\theta^0 = 2x^\mu dx^\mu, \tag{19}$$

$$\theta^1 = x_0 dx^1 - x_1 dx^0 + x_2 dx^3 - x_3 dx^2 \ (cyclic). \tag{20}$$

It is nondegenerate everywhere except at the origin where it vanishes.

The unique torsion-free connection form (compatible with θ) ω given in $\mathbf{R}^4 \setminus \{0\}$ by

$$\omega^{i0} = \omega^{jk} = -\rho^{-2}\theta^i \ (cyclic) \tag{21}$$

is flat and singular at $x = 0$. A regularization

$$\omega \mapsto \omega^{i0} = \omega^{jk} = -\phi(\rho^2)\theta^i,\ \rho^2 = \Sigma_\mu (x^\mu)^2, \tag{22}$$

results in a non-zero torsion

$$D\theta^i = (-\phi + \rho^{-2})(\theta^0 \wedge \theta^1 + \epsilon^{ijk}\theta_j \wedge \theta_k. \tag{23}$$

Thus we get a globally defined non-singular connection in an external $O(4)$ bundle. It induces space-time affine connection everywhere except at the origin. The induced connection has non-vanishing torsion.

Example 2.: Einstein and Rosen [9] considered two examples of solutions of (modified) vacuum field equations of general relativity with a degenerate metric. Let us show how both examples can be easily reformulated in terms of an $O(3,1)$ gauge theory. The first model discussed in [9] describes a uniformly accelerated frame in a flat Minkowski space. The second, described below, brings similar features with a non-flat connection. One takes here $M = \mathbf{R}^2 \times S^2$ with coordinates $x^0 = t, x^1 = u, x^2 = \psi, x^3 = \phi$, and the vierbein is given by

$$\begin{aligned}
\theta^0 &= u(u^2+2m)^{-1/2}dt, \\
\theta^1 &= 2(u^2+2m)^{1/2}du, \\
\theta^2 &= (u^2+2m)d\psi, \\
\theta^3 &= (u^2+2m)\sin(\psi)d\phi.
\end{aligned}$$

It degenerates at $u = 0$ (the "bridge"). The connection form can be represented by

$$\begin{aligned}
\omega^1{}_0 &= mu^{-1}(u^2+2m)^{-3/2}\theta^0, \\
\omega^2{}_1 &= u(u^2+2m)^{-3/2}\theta^2, \\
\omega^3{}_1 &= u(u^2+2m)^{-3/2}\theta^3, \\
\omega^3{}_2 &= \cot(\psi)(u^2+2m)^{-1}\theta^3,
\end{aligned}$$

At first sight it seems that ω is singular on the bridge in an analogy to (21). However, since the vierbein is degenerate, the 1–forms θ^a do not form a basis at $u = 0$ and therefore not every 1–form can be represented in terms of θ–s. In fact, we have

$$\omega^1 = m(u^2+2m)^2dt. \tag{24}$$

Torsion vanishes here, but ω is non–flat. This configuration (ω, θ) is a smooth solution of the vacuum (that is with right hand sides vanishing) field equations (8), (9) with $\lambda_1 = \lambda_3 = 0$.

7 Physical effects of a vanishing vierbein

We have seen that certain Lagrangians are smooth functions of configurations (ω, θ) even for degenerate θ–s, The requirement of smoothness is a strong selection criterion. In particular it rules out quadratic terms essential in theories with propagating torsion [15]. It has to be understood whether a singularity at $\det \theta = 0$ stabilizes a theory. Stability properties of metric theories of gravitation have been discussed by many authors [16]. Theories admitting vanishing vierbein need, however, special considerations. Hanson and Regge [1] speculated about a possibility of having a "torsion foam" – a region with vanishing vierbein and well defined principal connection. No dynamical mechanism which would make such configurations stable is known. Nevertheless it can be interesting to look for a possible method of detecting such a phenomenon. Here we discuss motion of a test particle which meets on its way a domain with vanishing vierbein.

Consider scattering of spinless particles on a double-cone shaped region where the vierbein vanishes (see Fig. 1). Equations of motion for a non-spinning test particle are (see [17])

$$\frac{d\pi^a}{dt} + {\omega_\mu}^a{}_b \frac{dx^\mu}{dt} \pi^b = 0, \tag{25}$$

$$\frac{dx^\mu}{dt} (\pi^a \theta^b_\mu - \pi^b \theta^a_\mu) = 0. \tag{26}$$

Notice that four-momentum π^a and four-velocity $\frac{dx^\mu}{dt}$ are a priori considered as independent variables. It is only through equation (26) that momentum becomes co-linear with velocity, but this co-linearity needs to obeyed at regular space-time only. Not inside of the vanishing vierbein domain.

In our case, in regions (I) and (III), where θ is invertible, momentum must be proportional to the velocity, as it follows from Eq. (26). In consequence trajectories γ_I and γ_{III} are geodesics. In region (II), where θ degenerates, momentum and velocity may be totally decoupled. Let (x_{ini}, π_{ini}) (resp. (x_{fin}, π_{fin})) denote the position and momentum of the particle when it enters (resp. leaves) vanishing–vierbein region (II). For given $x_{ini}, \pi_{ini}, x_{fin}, \pi_{fin}$ denote by $C(x_{ini}, \pi_{ini}, x_{fin}, \pi_{fin})$ the collection of all continuous paths $\gamma : t \mapsto \gamma(t)$ with the following property: π *when paralelly transported along* γ *from* x_{ini} *to* x_{fin} *coincides with* π_{fin} . Observe that a past trajectory γ_I, and the future trajectory γ_{III} are uniquely determined by x_{ini}, π_{ini} and x_{fin}, π_{fin} respectively. Observe also that two trajectories γ_{II} and $\gamma_{II'}$ both belong $C(x_{ini}, \pi_{ini}, x_{fin}, \pi_{fin})$ if and only if π_i is an eigenvector belonging to the eigenvalue zero of the flux of the curvature operator through the surface enclosed between γ_{II} and $\gamma_{II'}$.[5] Let $\rho(x_{ini}, \pi_{ini}; x_{fin}, \pi_{fin})$ be the number of elements (measure) of $C(x_{ini}, \pi_{ini}, x_{fin}, \pi_{fin})$. Would $\det \theta \neq 0$ everywhere inside region (II) then $\rho(x_{ini}, \pi_{ini}; x_{fin}, \pi_{fin}) = A\delta(x_{fin} - x_{fin0})\delta(\pi_{fin} - \pi_{fin_0})$, where x_{fin0} and π_{fin0} are uniquely determined by the initial data and the geometry. If $\theta = 0$ and $\Omega = 0$ in region (II) then $\rho(x_{ini}, \pi_{ini}; x_{fin}, \pi_{fin}) = \rho_1(x_{ini}, \pi_{ini}; x_{fin})\delta(\pi_{fin} - \pi_{fin0})$ where π_{fin0} is uniquely determined by the initial data. If so, then any observed non–zero dispersion of ρ implies degeneracy of θ in region (II) and reflects curvature effects in this region. Can some statistical effects that are normally attributed to quantum fluctuations be accounted for through the above mechanism?

The picture shown in Fig. 1 can be misleading in two ways. First, the time of emerging of the probe from region (II) is random. Second, a path of the particle should be continuous but need not be differentiable.
Fig. 2 shows a possible world line of a particle meeting on its way a "bubble of vanishing vierbein" in otherwise flat Minkowski space.

[5] For a non-Abelian Yang–Mills field as it is in our case a generalized flux should be used.

Figure 1: A possible trajectory of a test particle through region III with vanishing vierbein

Figure 2: Free particle meets on its way (event A) a bubble $\mathcal{O}$ of vanishing vierbein. Inside the bubble the particle has no compass to orient itself; its motion is random. Since the particle entered $\mathcal{O}$ with a future pointing momentum it gets reflected from the lower walls of the diamond and undergoes a succession of chaotic motions until it "chooses" to reach the upper cone. To an external observer the particle is "teleported in no time at all" from A to B.

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