

Event-Enhanced Quantum Theory and Piecewise Deterministic Dynamics

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The standard formalism of quantum theory is enhanced and definite meaning is given to the concepts of experiment, measurement and event. Within this approach one obtains a uniquely defined piecewise deterministic algorithm generating quantum jumps, classical events and histories of single quantum objects. The wave-function Monte Carlo method of Quantum Optics is generalized and promoted to the level of a fundamental process generating all the real events in Nature. The already worked out applications include SQUID-tank model and generalized cloud chamber model with GRW spontaneous localization as a particular case. Differences between the present approach and quantum measurement theories based on environment induced master equations are stressed. Questions: what is classical, what is time, and what are observers are addressed. Possible applications of the new approach are suggested, among them connection between the stochastic commutative geometry and Connes' noncommutative formulation of the Standard Model, as well as potential applications to the theory and practice of quantum computers.

I. INTRODUCTION

Quantum Mechanics occupies a very particular place among scientific theories; indeed it is at once one of the most successful and one of the most mysterious ones. Its success lies undoubtedly in the fact that using Quantum Mechanics one can predict properties of atoms, of molecules, of chemical reactions, of conductors and insulators and much more. These predictions were confirmed by precise measurements and by the technological progress that is based on quantum phenomena. The mystery resides in the problem of interpretation of Quantum Theory - which does not follow from the formalism itself but is left to discretion of a physicist. As a result, there is still no general agreement about how Quantum Mechanics is best understood and to what extent it can be considered as exact and complete.

As emphasized already by E. Schrödinger [1] what is definitively and completely missing in Standard Quantum Mechanics is an explanation of experimental facts, as it does not tell us how to generate time series of events recorded during real experiments on *single individual systems*. H.P. Stapp [2,3,4,5] and R. Haag [6,7] emphasized the role and importance of '*events*' in quantum physics. J. Bell [8] stressed the fundamental necessity of distinguishing '*definite events*' from '*just wavy possibilities*'.

In 1969 E.B. Davies [9] (see also [10]) introduced the '*space of events*' in his mathematical theory of quantum stochastic processes which extended the standard formalism of quantum theory. His theory went beyond a standard quantum measurement theory and, in its most general form, was not expressible in terms of quantum master equations alone. Later on Srinivas, in a joint paper with Davies [11], specialized Davies' general and mathematically sophisticated scheme to photodetection processes. Photon counting statistics predicted by this the-

ory were successfully verified in fluorescence experiments which caused R. J. Cook to revisit the question '*what are quantum jumps*' [12]. A related question: '*are there quantum jumps*' was asked by J. Bell [13] in connection with the idea of spontaneous localization put forward by Ghirardi, Rimini and Weber [14].

In the eighties quantum optics experiments started to call for efficient methods of solving quantum master equations that described effective coupling of atoms to the radiation modes. The works of Carmichael [15], Dalibard, Castin and Mølmer [16,17], Dum, Zoller and Ritsch [18], Gardiner, Parkins and Zoller [19], developed Quantum Monte Carlo (QMC) algorithm for simulating solutions of master equations.* The algorithm emerged from the seminal papers of Davies [9,10] on quantum stochastic processes, that were followed by numerous works on photon counting and continuous measurements (cf. Refs. [21,22,23]). It was soon realized (cf. e.g. [24,25,26,27,28]) that the same master equations can be simulated either by Quantum Monte Carlo method based on quantum jumps, or by a continuous quantum state diffusion. Wiseman and Milburn [29] discussed the question of which experimental detection schemes are better described by continuous diffusions rather than by discontinuous jump simulations. The two approaches were recently put into comparison also by Garraway and Knight [30], while Gisin et al. [31] argued that '*the quantum jumps can be clearly seen*' also in the quantum state diffusion plots. Apart from the numerical usefulness of quantum jumps and empirical observability of photon counts,

*Less general scheme was proposed by Teich and Mahler [20] who tried to extract a specific jump process directly from the orthogonal decomposition of time evolving density matrix.

the debate of their ‘*reality*’ continued. A brief synthesis of the present state of the debate has been given by Møller in the final paragraphs of his 1994 Trieste lectures [32]:

The macroscopic collapse has been explained, the elementary collapse, however remains as an essential and unexplained ingredient of the theory.

A real advantage of the QMC method: We can be sitting there and discussing its philosophical implications and the deep questions of quantum physics while the computer is cranking out numbers which we need for practical purposes and which we could never obtain in any other way. What more can we ask for?

In the present paper we argue that indeed ‘*more*’ can be not only asked for, but that it can be also provided. The picture that we propose developed from a series of papers [33,34,35,36,37,38,39,40,41], where we treated several applications including SQUID–tank [35] and cloud chamber model (with GRW spontaneous localization) [39,40,41]. In the sequel we will refer to it as Event Enhanced Quantum Theory (EEQT). EEQT is a minimal extension of the standard quantum theory that accounts for events. In the next three sections we will describe formal aspects of EEQT, but we will attempt to reduce the mathematical apparatus to the absolute minimum. In the final Sect. 4 we will propose to use EEQT for describing not only quantum measurement experiments, but all the real processes and events in Nature. The new formalism rises new questions, and in Sect. 4 we will point out some of these questions. One of the problems that can be discussed in a somewhat new light is that of the role of ‘observers’ and IGUS-es (using terminology of Gell–Mann and Hartle, cf. [42]). We will also make a comment on a possible interpretation of Connes’ version of the Standard Model as a stochastic geometry a’la EEQT, with jumps between the two copies of space–time. We will also mention relevance of EEQT to the theory and practice of quantum computers. The reader interested in the results and perspectives rather than in the mathematical formulation may skip Sections 2 and 3.

A. Summary

Using informal language EEQT can be summarized as follows: Given a ‘wavy’ quantum system Q we allow it to generate distinct classical traces - *events*. Quantum wave functions are not directly observable. They may be considered as hidden variables of the theory. Events are discrete and real. Typically one can think of detection events and pointer readings in quantum mechanics, but also of creation–annihilation events in quantum field theory. They can be observed but they do not need

an observer for their generation (although *some* may be triggered by observer’s participation). They are either recorded or they are causes for other events. It is convenient to represent events as changes of state of a suitable classical system. Thus formally we divide the world into $Q \times C$ – the quantum and the classical part. They are coupled together via a specific dynamics that can be encoded in a Liouville evolution equation for statistical states of the total $Q \times C$ system. To avoid misunderstanding we wish to stress it rather strongly: the fact that Q and C are coupled by a dissipative rather than unitary dynamics does not mean that noise, or heat, or chaos, or environment, or lack of knowledge, are involved. In fact each of these factors, if present – and all of them are present in real circumstances, only blurs out transmission of information between Q and C . The fact that Q and C must be coupled by a dissipative rather than by reversible dynamics follows from no–go theorems that are based on rather general assumptions [43,44,38]. We go beyond these abstract no–go theorems that are telling us what is not possible. We look for what is possible, and we propose a class of couplings that, as we believe, is optimal for the purposes of control and measurement. With our class of couplings no more dissipation is introduced than it is necessary for transmission of information from Q to C . Thus our Liouville equation that encodes the measurement process is to be considered as *exact*, not an approximate one (adding noise to it will make it approximate). Given such a coupling we show that the Liouville equation encodes in a unique way the algorithm for generating admissible histories of individual systems. This algorithm generalizes the one of Davies [9] as well as descending from the Davies’ theory wave-function Monte Carlo method.[†] The algorithm describes joint evolution of an individual $Q \times C$ system as a piecewise deterministic process. Periods of continuous deterministic evolution are interrupted by die tossings and random jumps that are accompanied by changes of state of C - events. We call it Piecewise Deterministic Process Algorithm, in short PDP (the term PDP has been introduced by M.H.A. Davis [45]). The algorithm is probabilistic what reflects the fact that the quantum world although governed by deterministic Schrödinger equation is, as we know from experience, *open* towards the classical world of events, and the total system $Q \times C$ is thus open towards the future. The PDP algorithm identifies the probabilistic laws according to which times of jumps and the events themselves are chosen. Our generalized framework en-

[†]The wave-function Monte Carlo method of Quantum Optics may be considered as mutilated version of the general Davies’ process. Indeed, Davies’ ‘events’ are forgotten there, and only the Liouville equation is accepted. That leads to arbitrariness of choosing between different jump or diffusion processes, arbitrariness which was not present in Davies’ original theory.

ables us not only to gain information about the quantum system but also to utilize it by a feed-back control of the $Q \times C$ coupling i.e. making the coupling dependent on the actual state of the classical system (which may depend on a record of previous events).

Briefly, our Event-Enhanced formalism can be described as follows: to define an *experiment* we must start with a division $Q \times C$. Assuming, for simplicity, that C has only finite number of states (which may be thought of as ‘pointer positions’) $\alpha = 1, \dots, m$, we define *event* as a *change of state* of C . Thus there are $m^2 - m$ possible events. An experiment is then described by a completely positive coupling V of Q and C . A coupling is specified by (i) a family H of quantum Hamiltonians H_α parametrized by the states of C , (ii) a family V of $m^2 - m$ of quantum operators $g_{\alpha\beta}$, with $g_{\alpha\alpha} \equiv 0$. In Refs. [33,34,35,36,37,38,39,40,41] we have described simple general rules for constructing $g_{\alpha\beta}$ -s, and we described non-trivial examples, including SQUID-tank model and generalized ‘cloud chamber’ model that covers GRW spontaneous localization model as a particular, homogeneous, case. The self-adjoint operators H_α determine the unitary part of quantum evolution between jumps, while $g_{\alpha\beta}$ determine jumps, their rates and their probabilities, as well as the non-unitary contribution to the continuous evolution between jumps. As an example, in the SQUID-tank model [46] the variable α is the flux through the coil of the classical radio-frequency oscillator circuit, and it affects, through a transformer, the SQUID Hamiltonian. $g_{\alpha\beta}$ have there also very simple meaning [35], as the shifts of the classical circuit momentum caused by a (smoothed out, operator-valued) quantum flux.

Time evolution of statistical states of the total $Q \times C$ system is described by the Liouville equation:

$$\dot{\rho}_\alpha = -i[H_\alpha, \rho_\alpha] + \sum_\beta g_{\alpha\beta} \rho_\beta g_{\alpha\beta}^* - \frac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \quad (1.1)$$

where

$$\Lambda_\alpha = \sum_\beta g_{\beta\alpha}^* g_{\beta\alpha}. \quad (1.2)$$

The operators H_α and $g_{\alpha\beta}$ can be allowed to depend explicitly on time, so that the coupling can be switched on and off in a controlled way. Moreover, to allow for phase transitions the quantum Hilbert space may change with α . We show in Sect. 2 that the above Liouville equation determines a piecewise deterministic process that generates histories of individual systems. In Sect. 3 we provide argument showing that within our framework the process is unique. Our PDP is given by the following simple algorithm which generalizes that of QMC:[‡]

[‡]Notice that in the H_α and $g_{\alpha\beta}$ in the algorithm may explicitly depend on time.

PDP Algorithm 1 *Let us assume a fixed, sufficiently small, time step dt . Suppose that at time t the system is described by a quantum state vector ψ and a classical state α . Compute the scalar product $\lambda(\psi, \alpha) = \langle \psi, \Lambda_\alpha \psi \rangle$. Then choose a uniform random number $p \in [0, 1]$, and jump if $p < \lambda(\psi, \alpha)dt$. When jumping, change $\alpha \rightarrow \beta$ with probability $p_{\alpha \rightarrow \beta} = \|g_{\beta\alpha}\psi\|^2 / \lambda(\psi, \alpha)$, and change $\psi \rightarrow g_{\beta\alpha}\psi / \|g_{\beta\alpha}\psi\|$. If not jumping, change*

$$\psi \rightarrow \frac{\exp\{-iH_\alpha dt - \frac{1}{2}\Lambda_\alpha dt\}\psi}{\|\exp\{-iH_\alpha dt - \frac{1}{2}\Lambda_\alpha dt\}\psi\|}, \quad t \rightarrow t + dt.$$

Repeat the steps.[§]

EEQT proposes that the PDP Algorithm describes *in an exact way* all real events as they occur in Nature, provided we specify correctly Q, C, H and V . More on this subject can be found in Sect. 4. In the following section we will formulate more precisely the basic structure of EEQT.

II. FORMAL SCHEME OF EEQT

Let us briefly describe the mathematical framework that we use. To define events, we introduce a classical system C , and possible events will be identified with changes of (pure) state of C . To concentrate on main ideas rather than on technical details we will consider the simplest situation corresponding to a finite set of possible events. It is possible and necessary in many applications to handle infinite dimensional generalizations of this framework. The space of states \mathcal{S}_c has m states, denoted by $\alpha = 1, \dots, m$. These are the pure states of C . Statistical states of C are probability measures on \mathcal{S}_c - in our case just sequences $p_\alpha \geq 0, \sum_\alpha p_\alpha = 1$. We will also consider the algebra of (complex) observables of C . This will be the algebra \mathcal{A}_c of complex functions on \mathcal{S}_c - in our case just sequences $f_\alpha, \alpha = 1, \dots, m$ of complex numbers. It is convenient to use Hilbert space language even for the description of that simple classical system. Thus we introduce m -dimensional Hilbert space \mathcal{H}_c with a fixed basis, and realize \mathcal{A}_c as the algebra of diagonal matrices $F = \text{diag}(f_1, \dots, f_m)$. Statistical states of C are then diagonal density matrices $\text{diag}(p_1, \dots, p_m)$, and pure states of C are vectors of the fixed basis of \mathcal{H}_c . Events are ordered pairs of pure states $\alpha \rightarrow \beta, \alpha \neq \beta$. Each event can thus be represented by an $m \times m$ matrix with 1 at the (α, β) entry, zero otherwise. There are $m^2 - m$ possible events. Statistical states are concerned with ensembles, while pure states and events concern individual systems.

[§]There are several methods available for efficient computation of the exponential for dt small enough - cf. Ref. [47].

Let Q be the quantum system whose observables are from the algebra \mathcal{A}_q of bounded operators on a Hilbert space \mathcal{H}_q . Its pure states are unit vectors in \mathcal{H}_q with understanding that proportional vectors describe the same quantum state. Its statistical states are given by non-negative density matrices $\hat{\rho}$, with $\text{Tr}(\hat{\rho}) = 1$. Then pure states can be identified with those density matrices that are idempotent $\hat{\rho}^2 = \hat{\rho}$, i.e. with one-dimensional orthogonal projections.

Let us now consider the total system $T = Q \times C$. First, we consider its statistical description, only after that we will discuss the dynamics and the coupling of Q and C . For the algebra \mathcal{A}_t of observables of T we take the tensor product of algebras of C and Q : $\mathcal{A}_t = \mathcal{A}_q \otimes \mathcal{A}_c$. Thus \mathcal{A}_t can be thought of as algebra of *diagonal* $m \times m$ matrices $A = (a_{\alpha\beta})$, whose entries are quantum operators: $a_{\alpha\alpha} \in \mathcal{A}_q$, $a_{\alpha\beta} = 0$ for $\alpha \neq \beta$. The classical and quantum algebras are then subalgebras of \mathcal{A}_t ; \mathcal{A}_c is realized by putting $a_{\alpha\alpha} = f_\alpha I$, while \mathcal{A}_q is realized by choosing $a_{\alpha\beta} = a\delta_{\alpha\beta}$. Statistical states of $Q \times C$ are given by $m \times m$ diagonal matrices $\rho = \text{diag}(\rho_1, \dots, \rho_m)$ whose entries are positive operators on \mathcal{H}_q , with the normalization $\text{Tr}(\rho) = \sum_\alpha \text{Tr}(\rho_\alpha) = 1$. Tracing over C or Q produces the effective states of Q and C respectively: $\hat{\rho} = \sum_\alpha \rho_\alpha$, $p_\alpha = \text{Tr}(\rho_\alpha)$. Duality between observables and states is provided by the expectation value $\langle A \rangle_\rho = \sum_\alpha \text{Tr}(A_\alpha \rho_\alpha)$.

We consider now dynamics. Quantum dynamics, when no information is transferred from Q to C , is described by Hamiltonians H_α , that may depend on the actual state of C (as indicated by the index α). We will use matrix notation and write $H = \text{diag}(H_\alpha)$. A *coupling* of Q to C is specified by a matrix $V = (g_{\alpha\beta})$, with $g_{\alpha\alpha} = 0$. To transfer information from Q to C we need a non-Hamiltonian term which provides a completely positive (CP) coupling. We propose to consider couplings for which the evolution equation for observables and for states is given by the Lindblad form:

$$\dot{A} = i[H, A] + \mathcal{E}(V^*AV) - \frac{1}{2}\{\Lambda, A\}, \quad (2.1)$$

$$\dot{\rho} = -i[H, \rho] + \mathcal{E}(V\rho V^*) - \frac{1}{2}\{\Lambda, \rho\}, \quad (2.2)$$

where $\mathcal{E} : (A_{\alpha\beta}) \mapsto \text{diag}(A_{\alpha\alpha})$ is the conditional expectation onto the diagonal subalgebra given by the diagonal projection, and

$$\Lambda = \mathcal{E}(V^*V). \quad (2.3)$$

We can also write it down in a form not involving \mathcal{E} :

$$\dot{A} = i[H, A] + \sum_{\alpha \neq \beta} V_{[\beta\alpha]}^* A V_{[\beta\alpha]} - \frac{1}{2}\{\Lambda, A\}, \quad (2.4)$$

with Λ given by

$$\Lambda = \sum_{\alpha \neq \beta} V_{[\beta\alpha]}^* V_{[\beta\alpha]}, \quad (2.5)$$

and where $V_{[\alpha\beta]}$ denotes the matrix that has only one non-zero entry, namely $g_{\alpha\beta}$ at the α row and β column. Expanding the matrix form we have:

$$\dot{A}_\alpha = i[H_\alpha, A_\alpha] + \sum_\beta g_{\beta\alpha}^* A_\beta g_{\beta\alpha} - \frac{1}{2}\{\Lambda_\alpha, A_\alpha\}, \quad (2.6)$$

$$\dot{\rho}_\alpha = -i[H_\alpha, \rho_\alpha] + \sum_\beta g_{\alpha\beta} \rho_\beta g_{\alpha\beta}^* - \frac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \quad (2.7)$$

where

$$\Lambda_\alpha = \sum_\beta g_{\beta\alpha}^* g_{\beta\alpha}. \quad (2.8)$$

Again, the operators $g_{\alpha\beta}$ can be allowed to depend explicitly on time.**

Following [40] we now define *experiment* and *measurement*:

Definition 1 *An experiment is a CP coupling between a quantum and a classical system. One observes then the classical system and attempts to learn from it about characteristics of state and of dynamics of the quantum system.*

Definition 2 *A measurement is an experiment that is used for a particular purpose: for determining values, or statistical distribution of values, of given physical quantities.*

Remark The definition of experiment above is concerned with the *conditions* that define it. In the next sections we will derive the PDP algorithm that simulates a typical *run* of a given experiment. In practical situations it is rather easy to decide what constitutes Q , what constitutes C and how to write down the coupling. Then, if necessary, Q is enlarged, and C is shifted towards more macroscopic and/or more classical. However the new point of view that we propose allows us to consider our whole Universe as ‘experiment’ and we are witnesses and participants of one particular run. Then the question arises: *what is the true C?* We will comment on this question in the closing section.

III. FROM THE LIOUVILLE EQUATION FOR ENSEMBLES TO THE PDP ALGORITHM FOR SINGLE SYSTEMS

A. Derivation of the PDP

Instead of constructing the PDP out of the Liouville equation, we will show that Eq. (2.6) is compatible with

**When $V = V^*$ i.e. when $g_{\alpha\beta} = g_{\beta\alpha}^*$, then our coupling satisfies the so called *detailed balance condition* - cf. [48,49].

the PDP Algorithm described in Section 1. Then, in the next subsection we will give arguments that can be used for proving its uniqueness. In order to prove compatibility of the Liouville equation (2.6) with the PDP Algorithm, i.e. to show that (2.6) follows from PDP by averaging, we will use the theory of piecewise deterministic processes (PDP) developed by M.H.A. Davis [45]. By Theorem (26.14) of Ref. [45] our PDP Algorithm leads to the following infinitesimal generator \mathcal{D} acting on complex valued functions $f(\psi, \alpha)^{\dagger\dagger}$

$$\begin{aligned} \mathcal{D}f(\psi, \alpha) = & \mathcal{Z}f(\psi, \alpha) + \\ & + \lambda(\psi, \alpha) \sum_{\beta} \int_{\phi} (f(\phi, \beta) - f(\psi, \alpha)) \mathcal{K}(d\phi, \beta; \psi, \alpha) , \end{aligned} \quad (3.1)$$

where

$$\mathcal{Z}f(\psi, \alpha) = \frac{d}{dt} f \left(\frac{\exp(-iH_{\alpha} - \frac{1}{2}\Lambda_{\alpha})\psi}{\|\exp(-iH_{\alpha} - \frac{1}{2}\Lambda_{\alpha})\psi\|}, \alpha \right) \Big|_{t=0}, \quad (3.2)$$

$$\lambda(\psi, \alpha) = \langle \psi, \Lambda_{\alpha} \psi \rangle, \quad (3.3)$$

and

$$\mathcal{K}(d\phi, \beta; \psi, \alpha) = \frac{\|g_{\beta\alpha}\psi\|^2}{\lambda(\psi, \alpha)} \delta \left(\phi - \frac{g_{\beta\alpha}\psi}{\|g_{\beta\alpha}\psi\|} \right) d\phi. \quad (3.4)$$

The above formula holds for *arbitrary* functions of ψ and α . However, because Q is quantum rather than classical, and because we are interested only in *linear* quantum mechanics, we need to consider only very special class of functions of ψ , namely those given by expectation values of linear quantum observables.^{††} To this end for each observable A of the total system we associate function $f_A(\psi, \alpha)$ defined by its expectation value: $f_A(\psi, \alpha) = \langle \psi, A_{\alpha} \psi \rangle$. Then, sandwiching the Liouville equation (2.6) between two ψ vectors, one can check (essentially by inspection) that its right hand side can be written up exactly as in Eq. (3.1) for $f = f_A$. That proves that our Liouville equation follows from the PDP Algorithm. Examples and details of the computation can be found in Refs [34,38,39].

B. Uniqueness of the PDP

In ordinary, i.e. non-enhanced by events, quantum theory there will be many random processes on the unit ball of the Hilbert space that reproduce the same master

equation for density matrix. The reason for this non-uniqueness being the fact that the convex set of statistical states of a quantum system is, contrary to the classical case, not a simplex. That is a given density matrix will decompose in infinitely many ways into pure states. (The fact that in a non-degenerate case there is a preferred orthogonal decomposition is just a mathematical artifact that has no statistical justification.) This non-uniqueness is equivalent to another fact, namely in quantum theory we have at our disposal not all functions $f(\psi)$ of pure states, but only those given by expectation values of linear observables $f_A(\psi) = \langle \psi, A_{\alpha} \psi \rangle$. The Liouville equation gives us time evolution, and thus its infinitesimal generator only on such functions - special polynomials in ψ of degree 2, while to reconstruct the random process in ψ space we need to know time evolution of characteristic functions of sets. Thus we have to extend our generator from functions $f_A(\psi)$ given by linear observables to arbitrary functions $f(\psi)$. Such an extension is non-unique and different extensions give rise to different random processes.

The situation is different when we discuss not arbitrary quantum master equations but experiments and measurements in EEQT. Here we have Q and C , and a special form of a Liouville equation - that given by Eq. (2.6). As we already remarked, it describes transfer of information from Q to C without introducing unnecessary (and harmful for the data) dissipation - that is why there should be zeros on the diagonal of the coupling V -matrix. That particular form of the Liouville equation has, as we will show now, a very special property. Namely, starting with a pure state (ψ, α) of the total system, after time dt we have a mixed state; there will be mixing along classical - which is uniquely decomposable, and there will be mixing along quantum - which decomposes nonuniquely. However, while mixing along classical is of the order dt , mixing along quantum is only of the order dt^2 . That is the special property that allows for a unique determination of the random process in infinitesimal steps. It is from this property that one can see again that our dissipation *is not caused by quantum noise* - rather it is the necessary minimal price that must be paid for any bit of information received from the quantum system.^{§§}

To see the last point explicitly, we use Eq. (2.7) to compute $\rho_{\alpha}(dt)$ when the initial state $\rho_{\alpha}(0)$ is pure:

$$\rho_{\alpha}(0) = \delta_{\alpha\alpha_0} |\psi_0\rangle \langle \psi_0|. \quad (3.5)$$

In the equations below we will discard terms that are higher than linear order in dt . For $\alpha = \alpha_0$ we obtain:

$$\begin{aligned} \rho_{\alpha_0}(dt) = & |\psi_0\rangle \langle \psi_0| - i[H_{\alpha_0}, |\psi_0\rangle \langle \psi_0|] dt - \\ & - \frac{1}{2} \{\Lambda_{\alpha_0}, |\psi_0\rangle \langle \psi_0|\} dt, \end{aligned} \quad (3.6)$$

^{††}If H or V explicitly depend on time, then we should add time t as the third argument of f .

^{††}As is well known, quantum mechanics can be considered as a particular case of classical mechanics, namely as (in general - infinite-dimensional) classical mechanics with a restricted set of observables.

^{§§}Quantum noise (cf. Ref. [50]), if present, it would appear on the diagonal of the $g_{\alpha\beta}$ matrix, and we have put it explicitly to zero.

while for $\alpha = \alpha_0$

$$\rho_{\alpha_0}(dt) = g_{\alpha\alpha_0}|\psi_0\rangle\langle\psi_0|g_{\alpha\alpha_0}^* dt \quad (3.7)$$

The term for $\alpha = \alpha_0$ can be written as

$$\rho_{\alpha_0}(dt) = p_{\alpha_0}|\psi_{\alpha_0}\rangle\langle\psi_{\alpha_0}|, \quad (3.8)$$

where

$$\psi_{\alpha_0} = \frac{\exp(-iH_{\alpha_0}dt - \frac{1}{2}\Lambda_{\alpha_0}dt)\psi_0}{\|\exp(-iH_{\alpha_0}dt - \frac{1}{2}\Lambda_{\alpha_0}dt)\psi_0\|}, \quad (3.9)$$

and

$$p_{\alpha_0} = 1 - \lambda(\psi_0, \alpha_0)dt. \quad (3.10)$$

The term with $\alpha \neq \alpha_0$ can be written as:

$$\rho_{\alpha}(dt) = p_{\alpha}|\psi_{\alpha}\rangle\langle\psi_{\alpha}|, \quad (3.11)$$

where

$$p_{\alpha} = \|g_{\alpha\alpha_0}\psi_0\|^2 dt, \quad (3.12)$$

and

$$\psi_{\alpha} = \frac{g_{\alpha\alpha_0}\psi_0}{\|g_{\alpha\alpha_0}\psi_0\|} \quad (3.13)$$

This representation is unique and it defines the infinitesimal version of our PDP.

IV. WHERE ARE WE NOW?

We have seen that Quantum Theory can be enhanced in a rather simple way to predict new facts and to streighten old mysteries. The EEQT that we have outlined above has several important advantages. One such advantage is of practical nature: we may use the algorithms it provides and ask computers ‘to crank out numbers that are needed in experiments and that can not be obtained in another way’. For example in [35] we have shown how to generate pointer readings in a tank circuit coupled to a SQUID, while in [39,40] the algorithm generating detection events of an arbitrary geometrical configuration of particle position detectors was derived. As a particular case, in a continuous homogeneous, limit we have reproduced GRW spontaneous localization model. Many other examples come from quantum optics, since QMC is a special case of our approach, namely when *events* are not feed-backed into the system and thus do not really matter.

Another advantage of EEQT is of a conceptual nature: in EEQT we need only one postulate: *that events can be observed*. All the rest can and should be derived from this postulate. All probabilistic interpretation, everything that we have learned about eigenvalues, eigenvectors, transition probabilities etc. can be derived from

the formalism of EEQT. Thus in [33] we have shown that probability distribution of eigenvalues of Hermitian observables can be derived from the simplest coupling, while in [41] we have shown that Born’s interpretation can be derived from the simplest possible model of a position detector. Moreover, in [38] it was shown that EEQT can also give definite predictions for non-standard measurements, like those involving noncommuting operators.

It is also possible that using the ideas of EEQT may throw a new light into some applications of non-commutative geometry. Namely, when C consists of *two points*, then our V can be interpreted as Quillen’s superconnection (cf. [51] and references there). Indeed, with $g_{10} = \phi M, g_{01} = \bar{\phi} M^\dagger$, our V of Section 2 plays the same role as D_{∇} operator in Connes’ noncommutative gauge theory [52, Section 2]. That suggests that Connes’ Z^2 -graded non-commutative geometry version of the Standard Model can be interpreted and understood as a *commutative but stochastic geometry*, with continuous parallel transport (determined by gauge fields) interrupted by random jumps between two copies of the universe (determined by Higgs fields), as in the PDP algorithm.***

Another potential field of application of EEQT is in the theory and practice of quantum computation. Computing with arrays of coupled quantum rather than classical systems seems to offer advantages for special classes of problems (cf. [54] and references therein). Quantum computers will have however to use classical interfaces, will have to communicate with, and be controlled by classical computers. Moreover, we will have to understand what happens during individual runs. Only EEQT is able to provide an effective framework to handle these problems. It keeps perfect balance of probabilities without introducing ‘negative probabilities’, and it needs only standard random number generators for its simulations. For a recent work where similar ideas are considered cf. [55]

EEQT is a precise and predictive theory. Although it appears to be correct, it is also yet incomplete. The enhanced formalism and the enhanced framework not only give enhanced answers, they also invite asking new questions. Indeed, we are tempted to consider the possibility that PDP can be applied not only to what we call experiments, but also, as a ‘world process’ to the entire universe (including all kinds of ‘observers’). Thus we may assume that all the events that happened were generated by a particular PDP process, with some unknown Q, C, H and V . Then, assuming that past events are known, the future is partly determined and partly open. Knowing Q, C, H, V and knowing the actual state (even if this knowledge is fuzzy and uncertain), we are in position to use the PDP algorithm to generate probable future series of events. With such a promotion of the

***Cf. also [53] for relation between superconnections and classical Markov processes.

PDP to the role of a universal world process questions arise that could not be asked before: *what is C and what is V?*, and perhaps also: *what is t?* and *what are 'we'*. Of course we are not in a position to provide answers. But we can discuss possibilities and we can provide hints.

A. What is time?

Let us start with the question: *what is time?* Answering that time is determined by the thermodynamic state of the system [56] is not enough, as we would like to know how did it happen that a particular thermodynamic state has evolved, and to understand this we must assume evolution, and thus we are back with the question: what is time if not just counting steps of this evolution. We are tempted to answer: *time is just a measure of the number of events that happened in a given place*. If so, then time is discrete, and there is another time, that counts the deterministic steps between events. In that case die tossing to decide whether the next step is to be an event or not is probably uneconomic and unnecessary; it is quite possible that the Poissonian character of events is a result of some ergodic theorem, when we use not the 'true' discrete time, but some continuous 'averaged' time (averaged over a neighborhood of a given place). Thus a possible algorithm for a finite universe would be discrete, with die tossing every N steps, N being a fixed integer, and continuous, averaged time would appear only in a thermodynamic limit. In fact, in a finite universe, die tossing should be replaced by a deterministic algorithm of sufficient complexity. A spectrum of different approaches to the problem of time, some of them similar to the one presented above, can be found in Ref. [57]. In a recent paper J. Schneider [58] proposes that a passing instant is the production of a meaningful symbol, and must be therefore formalized in a rigorous way as a transition. He also states that the linear time of physics is the counting of the passing instants, that time is linked with the production of meaning and is irreversible per se. We agree only in part, as we strongly believe that physical events, and the information that is gained due to these events, are objective and primary with respect to secondary mental or semantic events.

B. What is classical?

We consider now the question: *what is classical?* In each practical case, when we want to explain a given phenomenon, it is clear what constitutes *events* for us that we want to account for. These events are classical, and usually we can safely extend the classical system C towards Q gaining lot and loosing little. But here we are asking not a practical question, we are asking a fundamental question: what is *true C*? There are several possibilities

here, each one having advantages and disadvantages, depending on circumstances in which the question is being asked. If we believe in quantum field theory and if we are ready to take its lesson, then we must admit that one Hilbert space is not enough, that there are inequivalent representations of canonical commutation relations, that there are superselection sectors associated to different phases. In particular there are inequivalent infrared representations associated to massless particles (cf. [59] and references therein). Then classical events would be, for instance, soft photon creation and annihilation events. That idea has been suggested by Stapp [4,60] some ten years ago, and is currently being developed in a rigorous, algebraic framework by D. Buchholz [61,62].

Another possibility is that not only photons, but also long range gravitational forces may take part in the transition from potential to the actual. That hypothesis has been expressed by several authors (see e.g. contributions of F. Károlyházy et al., and R. Penrose in [64]; also L. Diosi [27]). The two possibilities quoted above are not satisfactory when we think of a finite universe, evolving step by step, with a finite number of events. In that case we do not yet know what is gravity and what is light, as they, together with space, are to emerge only in the macroscopic limit of an infinite number of events. In such a case it is natural to look for C in Q . We could just *define* event as a nonunitary change of state of Q . In other words, we would take for \mathcal{S}_c the only available set - the unit ball of the Hilbert space. This possibility has been already discussed in [38]. That choice of \mathcal{S}_c is also necessary when we want to discuss the problem of objectivity of a quantum state. If quantum states are objective (even if they can be determined only approximately), then the question: 'what is the actual state of the system' is a classical question - as an attempt to quantize also the position of ψ would lead to a nonsense. We should perhaps remark here that our picture of a fixed Q and fixed C that we have discussed in this paper is oversimplified. When attempting to use the PDP algorithm to create a finite universe in the spirit of space-time code of D. Finkelstein (cf. [63] and references therein), or bit-string universe of P. Noyes (cf. Noyes' contribution to [64]) we would have to allow for Q and C to grow with the number of events. Our formalism is flexible enough to adjust to such a change.

C. What is V?

The next question that we have asked is *what is V?* . To answer this question we must first know the answers to the two previous questions. In practical situations, when C is specified, then V is chosen so that it provides the best fit to the experimental data. There are simple rules to construct V and we have discussed in details several explicit examples in the already quoted references. On the other hand, when C is related to the infrared

representations - we do not know the answer yet, but we can see several ways of attacking this problem, and we hope to return to this case in the future. When Q is finite and \mathcal{S}_c is the unit ball in the Hilbert space - so that we deal with a ‘natural and universal’ C , then there is also a natural and universal V . Indeed, an event is then simply a pair of state vectors, $|\psi\rangle, |\psi'\rangle$, and to such a pair we can canonically associate the operation $g_{\psi\psi'} = |\psi\rangle\langle\psi'|$. That natural choice defines V up to a numerical coupling constant. We remark that in this case \mathcal{S}_c is infinite and continuous, so that the simplified mathematical framework that we have outlined is insufficient and must be extended. That this can be easily done was demonstrated in [35,38]. In the continuous case the diagonal of the V matrix is of measure zero and as such - unimportant. But the conditional expectation \mathcal{E} of Sect. 2 must be regularized. It is to be however remarked that what is *natural* from pure mathematical point of view, is usually oversimplified or wrong when applied to a physical problem. Therefore in any practical problem the universal C is too big, and the natural V is too simple.

D. Dynamics and Binamics

Having provided tentative answers to some of the new questions, let us pause to discuss possible conceptual implications of the EEQT. We notice that EEQT is a dualistic (and even syncretistic) theory. In fact, we propose to call the part of time evolution associated to V by the name of *binamics* - in contrast to the part associated to H , which is called *dynamics*. While dynamics deals with the laws of exchange of forces, binamics deals with the laws of exchange of bits (of information). We believe that these two sets of laws refer to different projections of one reality and neither one of these projections can be completely reduced to another one. Moreover, concerning the reality status, we believe that ‘bits’ are as real as ‘forces’. That this is indeed the case should be clear if we apply the famous Lande’s criterion of reality: real is what can kick. We know that information, when applied in an appropriate way, may cause changes and may kick - not less than a force.

E. What are we ?

We have used the term ‘we’ too many times to leave it without a comment. Certainly *we* are partly Q and partly C (and partly of something else). But not only we are subjects and spectators - sometimes we are also actors. In particular we can *gain and utilize information* [42]. How can this happen? How can we control anything? Usually it is assumed that we can prepare states by manipulating Hamiltonians. But that can not

be exactly true. We are not in power to change coupling constants or Hamiltonians that are governing fundamental forces of Nature. And when we say that we can manipulate Hamiltonians, we really mean that we can manipulate *states* in such a way that the standard fundamental Hamiltonians act on these special states *as if they were* phenomenological Hamiltonians with classical control parameters and external fields that we need in order to explain our laboratory procedures. So, how can we manipulate states without being able to manipulate Hamiltonians? We can only guess what could be the answer of other interpretations of Quantum Theory. Our answer is: we have some freedom in manipulating C and V . We can not manipulate dynamics, but binamics is open. It is through V and C that we can feedback the processed information and knowledge - thus our approach seems to leave comfortable space for IGUS-es. In other words, although we can exercise little if any influence on the continuous, deterministic evolution^{†††}, we may have partial freedom of intervening, through C and V , at bifurcation points, when die tossing takes place. It may be also remarked that the fact that more information can be used than is contained in master equation of standard quantum theory, may have not only engineering but also biological significance. In particular, we provide parameters (C and V) that specify event processes that may be used in biological organization and communication. Thus in EEQT, we believe, we overcome criticism expressed by B.D. Josephson concerning universality of quantum mechanics [65,66] . The interface between Quantum Physics and Biology is certainly also concerned with the fact that a lot of biological processes (like the emergence of naturally catalytic molecules or the evolution of the genetic code) can be in principle described and understood in terms of physical quantum events of the kind that we have discussed above.

We believe that are our proposal as outlined in this paper and elaborated on several examples in the quoted references is indeed the minimal extension of quantum theory that accounts for events. We believe that, apart of its practical applications, it can also serve as a reminder of existence of new ways of looking at old but important problems.

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