On Quantum Jumps, Events and Spontaneous Localization Models
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Abstract

We propose a precise meaning to the concepts of "experiment", "measurement" and "event" in the event-enhanced formalism of quantum theory. A minimal piecewise deterministic process is given that can be used for a computer simulation of real time series of experiments on single quantum objects. As an example a generalized cloud chamber is described, including multiparticle case. Relation to the GRW spontaneous localization model is discussed.

1   Events, Measurements and Experiments

In his paper "The Philosophy of Experiment" E. Schrödinger[1] wrote:

> "The new science (q.m.) arrogates the right to bully our whole philosophical outlook. It is pretended that refined measurements which lend themselves to easy discussions by the quantum mechanical formalism could actually be made. (...) Actual measurements on single individual systems are never discussed in this fundamental way, because the theory is not fit for it.(...) We are also supposed to admit that the extent of what is, or might be, observed coincides exactly with what the quantum mechanics is pleased to call observable."

It is well known to every experimentalist that more can be, and *is* being observed than q.m. is pleased to call an observable. What Schrödinger asserted 40 years ago is even more valid today. Actual measurements are nowadays often being made on single quantum objects. They give us finite time series of *events*. The importance of this concept of an "event", and the intrinsic

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incapability of quantum theory to deal with it, have been stressed by several authors. H.P. Stapp[2,3,4] emphasized the role of "events" in the "world process". G. F. Chew used Stapp's[5] ideas on soft–photon creation–annihilation processes and proposed[6] the term "explicate order", complementing Bohm's "implicate" quantum order, to denote the world process of "gentle" creation–annihilation events. A. Shimony[7] accentuated "that quantum mechanics does not have, in any obvious way, the conceptual tools for explaining how potentialities are actualized". He thought a solution in refining the concept of event and conjectured that "the actualization of potentiality must not be conceived as a limiting case of probability - as probability 1 or 0. Instead, actuality and potentiality are radically different modalities of reality". R. Haag[8] emphasized that "an event in quantum physics is discrete and irreversible" and that "we must assume that the arrow of time is encoded in the fundamental laws ...". He also suggested[9] that "transformation of possibilities into facts must be an essential ingredient which must be included in the fundamental formulation of the theory".[1]

J.S. Bell[11,11a] reprimanded the misleading use of the term "measurement" in quantum theory. He opted for banning this word from our quantum vocabulary, together with other vague terms such as "macroscopic", "microscopic", "observable" and several others. He suggested to replace the term "measurement" by that of "experiment", and also not to speak of "observables" (the things that seem to call for an "observer") but to introduce instead the concept of "beables"- the things that objectively "happen–to–be (or not–to–be)".[2] But there is no place for "events" or for "beables" in ordinary quantum theory. That is so because each "event" must have three characteristic features:

- it is classical,

- it is discrete,

- it is irreversible.

If just one of these three features is relaxed, then what we have is not yet an "event".

It must be *classical*, because it must obey to the classical "yes-no" logic; it must never be in a "superposition" of it being happened and being unhappened. Otherwise it would not be an event.

It must be *discrete*. It must happen wholly. An event that "approximately"

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[1]A.O. Barut[10] seeked also a theory of events, but in terms of "wave lumps" rather than in terms of discrete collision–like or detection–like discrete occurences

[2]Calling observables "observables" can be, however, justified in the event-enhanced formalism that we are outlining here.

happened is not an event at all.

It must be *irreversible*, because it can not be made "undone". This feature distinguishes real evnts from the "virtual" ones. Once something happened – it happened at a certain time instant. It must have left a trace. Even if this trace can be erased, the very act of erasing will change the future – not the past. Something else may happen later, but it will be already a different event. We believe that *the events, and nothing but events, are pushing forward the arrow of time.*

Once these three characteristics of an event are accepted (and they are evident for an experimentalist), it becomes clear what is necessary if we want to enhance the standard formalism of quantum theory so as to include events into it.

First – we must allow the formalism to include *classical* quantities. We believe that is better when this is done openly rather than via a back door. Second – we must allow for an *irreversible* coupling between quantum and classical degrees of freedom. It must be stressed that the minimal irreversibility that we are talking about here *is not* the result of "noise" "chaos" or "environment", but is forced upon by a universal law: the one stating that: *information must be paid with dissipation.*

But that is not enough. We need to make another important step: we must learn how to describe *finite time series of events* that are registered in experiments. It is out of such time series that expectation values can be computed. But the experimentalist is interested not only (or not at all) in expectation values. (He may be, for instance, interested in time correlations for a finite sequence of neutron detection events[12,13].) Moreover, as human beings, we are interested perhaps only in one such time series of events at all – as we cannot enter twice into the same stream of time. Not only we want to be able to compute statistical characteristics of ensembles. We also want to be able to simulate on digital computers experimental finite time series for individual systems. We want to be able to simulate the events that form up "mini-universes" governed by the same laws as that great one that we live in. We want to know how to account for all regularities that are apparently seen in the acquired data. We also seek guiding principles that will tell us what to do if we want to see still more regularities in the workings of Nature that are being unveiled to us.

Can quantum theory be tailored so as to suit these demands? Or, perhaps, it suits them already? It is not our aim to describe all the efforts that has been taken by quantum physicists in last 70 years in this direction. Scanning through the recent physical literature will give the reader an idea of what was done, what progress has been achieved. In this paper we just want to describe what seems to us to be *the minimal enhancement* of quantum theory

that suits the demands of human experience and technology. It has been developed in a series of papers by Ph.Blanchard and the present author[14−21]. It is one of the aims of the present paper to express, in a condensed form, the "philosophical backbone " that can be felt through the mathematical skeen of the several model discussed in Refs (14-21). From the structural and from mathematical point of view, the three most essential features of this enhanced formalism are:

- tensoring of non–commutative quantum algebra with a commutative algebra of functions,

- replacing Schrödinger's unitary dynamics of pure states with a suitable completely positive time evolution of families of density matrices,

- interpreting the continuous time evolution of statistical states of the total (i.e. quantum+classical) system in terms of a piecewise – deterministic random process on pure states. The process consists of pairs (quantum jump,classical event) interspersed by random periods of Schrödinger-type, in general non–linear, continuous evolution.

In an informal way that extension can be understood as follows. When describing any real (i.e. actual, not some imagined one) quantum system Q, we always have to decide at which point our quantum description ends. We cannot include "everything" into Q. That for the simple reason that we do not know what "everything" is (tomorrow we will perhaps know more than we know today). Then, where Q ends, the new territory begins. We may ignore this new territory completely and restrict ourselves to Q. Or, we can try to take it also into account. This new territory may include "environment", may include "events", but it may include also "us" - who are creating theories, making predictions, and who are checking those theories by doing experiments. Taking into account environment is standard and does not need a separate discussion. On the other hand discussing human minds and acquisition of human knowledge would take us too far, and to a shaky ground. The present version of the formalism, even after "enhancement" as sketched in the present paper, does not seem to be powerful enough to carry out this last step. So, we restrict ourselves to the decription of events. The logic that we are necessarily using when checking our reasoning against errors, and when describing experimental events – is the classical logic of Aristotle and Boole. This classical description can, and always partially will, extend downwards: from minds to brains, to senses, to communication channels, and still down to measuring devices. What results from such an extension constitutes a classical system C. Of course we may and we should use, whenever it helps, the knowldge that we have about microscopic structure of instruments, light quanta and brains, and we may appropriately enlarge Q. But in each case

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there will be a limit. Going beyond that limit will be useless and often impractical if our aim is to understand a given physical phenomenon rather than to construct an abstract "theory of everything" that computes and explains nothing. Anyhow, whatever we choose to include into Q, at some point we will have to decide what constitutes for us the "events" that we want to account for. It is the believe of the present author that *an event can be only defined as "a change of a state of C"*. As an example we may think of Q as being a single particle, for instance an electron, and of C as a particle detector that can be in one of its two states: "off" (particle undetected) or "on" (particle detected). Then an "experiment", or a "measurement", is nothing but an appropriate coupling of Q and C together, observing C, and learning from it about Q. Or, going to the extreme end, we can think of Q including "everything" but "minds of the observers" [22]. As noticed by Heisenberg[23], denying necessity of such a cut, and attempting to make all the universe into a single quantum system, leads to a situation when "physics has vanished, and only mathematical scheme remains". We would go even further and add: but even not that, because this mathematical scheme is then also a part of the quantum universe, and so it is only one of the unimaginable infinity of other potentially possible schemes.

On the practical side, we believe that our approach, after suitable translation, can be acceptable even by the quantum purists who probably would like to deny the fact that there are any *classical* events. First of all let us point it out that they are using, perhaps without noticing it, classical events in the form of *position of the state vestor in the space of all possible state vectors*. An attempt of "quantizing" even this position would lead to an absurd infinite sequence of quantizations, and thus to a theory that looses completely contact with reality. So, even if "wave function is observable, after all", then *it is a classical observable*. Second, quantum purists may consider our C as a copy of their "pointer basis". Then perhaps they will notice that our dynamics always respects this basis. It is for this reason that we may call it consistently classical. They will also notice that in our approach, it is only in special cases that the effective evolution of reduced quantum density matrix separates – which makes our approach more general. Finally, we do not mind calling our approach "phenomenological" or "effective". In fact we believe that any theory that has anything to do with the particular reality that is given to us, must be both phenomenological and effective to a necessary degree.

Let us discuss now the two terms: "measurement" and "experiment". The term "experiment" seems to be less dangerous than that of "measurement" – the last one being discredited owing to its over–fuzziness and arbitrariness.[10,11] Moreover, from any actual coupling between a quantum system and measuring and recording devices, from any actual experiment, we can

usually learn many different things by postselection of data and by numerical analysis of the data afterwards.[12,13] As it was stressed by E. Schrödinger in the opening quotation, an actual experiment can rarely – if ever – be considered as a measurement of a quantum mechanical "observable". According to already quoted reprimanding papers by J. Bell, the term "observable" should be banned – together with that of "measurement" and a couple of others. We feel that rather than ban, it is better, whenever possible, to define the terms precisely. We propose the following definitions:

Definition 1 *An* **experiment** *is a CP coupling between a quantum and a classical system. One observes then the classical system and attempts to learn from it about characteristics of state and of dynamics of the quantum system.*

Definition 2 *A* **measurement** *is an experiment that is used for a particular purpose: for determining values, or statistical distribution of values, of given physical quantities.*

By *CP coupling* we mean a two–parameter family $\phi_{t,s}$, $t \leq s$, of completely positive maps on the algebra of $C + Q$, satisfying the composition law

$$\phi_{t,s}\phi_{s,r} = \phi_{t,r}, \tag{1}$$

and

$$\phi(t,t) = Id \tag{2}$$

If ϕ_t acts on operators, then speaking of a "measurement", we will usually require

$$\phi_{t,s}(I) = I. \tag{3}$$

Remark 1. We can also think of experiments that include state preparation (and thus selection or filtering) procedures. In that case the condition (3) should be relaxed. In the present paper we will not discuss state preparation parts of experiments, so we will always assume Eq. (3) to hold.

Remark 2. When speaking of experiment and of observing the classical system, we may have in mind observations on a single system that last in time, or we can allow for repeated observations on similarly prepared systems. The standard quantum mechanical concept of measurement, whose analysis goes back to J. von Neumann, deals exclusively with the second case, while our description allows for experiments on single systems. That such an extension is necessary is nowadays clear, as advances in technology make such experiments on quantum systems more and more frequent. Moreover, we are finding pure quantum effects in macroscopic systems, and there we are making prolonged experiments on single objects (e.g. SQUID-s). Finally,

in some experiments, as for example for a SQUID-tank system[24], we may be changing the quantum dynamics depending on the actual state of our measuring devices. The event–enhanced formalism easily accomodates such experiments.[18]

Remark 3. One more important remark is due here. In the definition above we took it for granted that the classical system C can be "observed", and that we do not have to define what does *that* mean. The point is that states of a quantum system always change as a result of an observations. It is for this reason that a "quantum measurement theory" is necessary. On the other hand, given two classical systems C_1 and C_2, we can couple C_1 and C_2 together in such a way that the information about the actual state of C_1 is translated into the actual state of C_2, and that the change of the actual state of C_1 caused by this coupling is arbitrarily small. In fact, for a complete consistency, we need something more: we must prove that if C_1 is coupled to Q, and C_2 is coupled to C_1, then this latter coupling can be such that it will not affect Q otherwise than via C_1. Such a proof may be difficult or impossible if one wants to think of C as of, for instance, a classical electromagnetic or gravitational field.

2 The Minimal Extension of Quantum Theory that Accounts for "Events"

The algebraic framework that is needed for a mathematical formulation of such an extension of quantum theory is described in Refs. (14,19). Here we will describe it briefly and in plain terms, without aiming at a mathematical rigour. A formulation using more general, algebraic, language is also possible, but no significant physical insight would be gained concerning the problems at hand.

Suppose that we want to describe a quantum system Q coupled to a classical system C. Let $\mathcal{H}_q$ be the Hilbert space which is used to describe Q. Pure states of Q are described by unit vectors $\psi \in \mathcal{H}_q$ (modulo phase). Statistical states are described by density matrices $\hat{\rho}$, $\hat{\rho} \geq 0$, $\mathrm{Tr}\,(\hat{\rho}) = 1$. Observables are described by operators $A \in \mathcal{H}_q$, and their expectation values in states are given by $< A >_\psi = (\psi, A\psi)$, $< A >_{\hat{\rho}} = \mathrm{Tr}\,(A\hat{\rho})$.[3]

Classical system C is described by a space $\mathcal{S}_{cl}$, whose points are pure states of C. For a dynamical system, $\mathcal{S}_{cl}$ will usually be the phase space of C. But, if the dynamics of C itself is trivial (as it will be, for instance, if C is

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[3]We should stress here that the traditional term "expectation value" can be justified within event-enhanced formalism. It is indeed an expectation value computed from a sequence of events that result from a special coupling of Q to appropriate C.

discrete), then $\mathcal{S}_{cl}$ can be the configuration space of C as well. In general $\mathcal{S}_{cl}$ should be equipped with some measurable structure. To make our discussion as simple as possible, and not to dissipate our attention on mathematical subtleties, let us assume here that $\mathcal{S}_{cl}$ is a finite set, parameterized by a parameter ι. Thus we will skip free dynamics of C. [4] Statistical states of C are probabilistic measures on $\mathcal{S}_{cl}$ or, in our case, sequences of non-negative numbers μ_ι, with $\sum_\iota \mu_\iota = 1$. Observables of C are functions on $\mathcal{S}_{cl}$.

We consider now the total system $Q + S$. Its pure states are pairs (ψ, ι). Its statistical states are families ρ_ι, such that, for each ι, ρ_ι is a positive operator in $\mathcal{H}_q$, and that $\sum_\iota \mathrm{Tr}\,(\rho_\iota) = 1$. A statistical state $\{\rho_\iota\}$ of $C + Q$ determines, by partial tracing, the effective states $\hat{\rho}$ of Q and μ of C: $\hat{\rho} = \sum_\iota \rho_i$, $\mu_\iota = \mathrm{Tr}\,(\rho_\iota)$. Observables of $Q + C$ are families A_ι. Expectations are given by $< A >_\rho = \sum_\iota \mathrm{Tr}\,(A_\iota \rho_\iota)$. The allowed couplings of Q and C are described by the Liouville equation with a (possibly time–dependent) Lindblad–type generator[25,26,27]

We will not need the most general form of such a coupling here, so we describe only a particular, simple case. It will correspond to the following *informal* description of the coupling:

- the coupling does not explicitly depend on time (it is easy to relax this condition)

- there is a certain number of "quantum properties" F_α, $\alpha = 1, 2, \ldots$ of Q that we want to discriminate between (to "measure"),

- if the classical system is in a state ι then the quantum system evolves according to a Hamiltonian quantum dynamics described by some quantum Hamiltonian H_ι,

- for each α there is a certain transformation of $\mathcal{S}_{cl}$ with the following meaning: if the quantum system "has" a property F_α while the classical system is in a state ι, then C switches from ι to a new state, which we denote $\alpha(\iota)$.

For simplicity we will assume that the maps $\alpha : \iota \mapsto \alpha(\iota)$ are one-to-one. Even more, for the present purpose we will assume that each of them is an involution, that is that $\alpha(\alpha(\iota)) = \iota$. That assumtions corresponds to the idea that C consists of two-state subsystems, and that each α is flipping states in some of these subsystems. Or, in other words, that the classical (pure) states are described by strings of zero-one bits, and that every transformation α flips the bits in some (depending on α) substring. Thus our coupling –

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[4]Taking it into account is not a problem. In the SQUID–tank model[18] $\mathcal{S}_{cl}$ is a two-dimensional phase space with dumped oscillator dynamics.

although (because of no-go theorems) necessarily irreversible – is only as much irreversible as demanded by its very nature. The reversible maps α resemble thus the idea of conservative logic[28].

We have not yet described what we mean by the term "property" of a quantum system. The simplest example of a property is an orthogonal projection $e \in \mathcal{H}_q$. A *fuzzy* property is a positive operator a, $0 \leq a \leq I$. It is convenient to allow for properties to have different "intensivities", therefore we relax the assumption $a \leq I$, and so a can be just a positive operator. Thus, what we call a "property", corresponds to what is also known, especially in the Ludwig's school, under the name "effect". In application F_α-s will either come from some spectral measure, or from some smoothed-out spectral measures (so called POV measure) or, as will be the case in the next section, from smoothed integrals of number density operator over space regions. Finally, in order to allow for still wider applications of the formalism, we need not even to assume that the operators F_α are Hermitian (although in most applications they will be positive) - then they may be called "operations". [5]

With this heuristic description of the coupling in mind, we will write now the form of the Liouville equation that corresponds to it. A mathematical justification can be found in Ref. (15,19); see also references therein.

$$\frac{d\rho_\iota}{dt} = -i\,[H_\iota, \rho_\iota] + \sum_\alpha F_\alpha \rho_{\alpha(\iota)} F_\alpha^\star - \frac{1}{2}\{\Lambda, \rho_\iota\}, \tag{4}$$

where we have denoted

$$\Lambda \doteq \sum_\alpha F_\alpha^\star F_\alpha \ . \tag{5}$$

We can always switch the time evolution between states and observables by using the duality equation: $< A(t) >_\rho = < A >_{\rho(t)}$. Then for observables we have almost the same equation as above, except for the sign in front of the commutator and the order of F and $F^\star$ in the second term:

$$\frac{dA_\iota}{dt} = i\,[H_\iota, A_\iota] + \sum_\alpha F_\alpha^\star A_{\alpha(\iota)} F_\alpha - \frac{1}{2}\{\Lambda, A_\iota\}. \tag{6}$$

We notice now that in a special case, *if the quantum Hamiltonian does not depend on the state of the classical system*, i.e. if $H_\iota \equiv H$ for each ι, then Eq. (4) can be summed up over ι. This is so because we have assumed that each of the transformations α is one–to–one and onto (i.e. it is a permutation in $\mathcal{S}_{cl}$). Therefore, for each α, $\sum_\iota \rho_{\alpha(\iota)} = \sum_\iota \rho_\iota = \hat{\rho}$. It follows that, in this

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[5]Usually, when modelling a measurement, the F_α are positive, but there are experiments that are not measurements (as for instance the experiment performed by Nature that we are participating in). Then F_α may be creation or annihilation operators.

special case, the time evolution for the effective quantum states separates and we have:

$$\frac{d\hat{\rho}}{dt} = \sum_\alpha F_\alpha \hat{\rho} F_\alpha^\star - \frac{1}{2}\{\Lambda, \hat{\rho}\}. \tag{7}$$

It should be stressed that this separating property of the Liouville equation that governs time evolution of the total system $C + Q$, need not to hold for more general models. If the quantum Hamiltonian depends on ι, or if there is a non-trivial dynamics in the classical system, then summing over ι-s as above is not possible. That is what happens e.g. in the SQUID-tank model. We also notice that, even in those cases when the effective quantum dynamics separates, Eq.(7) contains less information than Eq.(4). Indeed, information about the transformations $\alpha : \iota \mapsto \alpha(\iota)$, that is information about events, is lost. Therefore there can be many couplings of Q to other systems (classical or quantum) that determine effectively the same evolution of statistical quantum states. Some of these couplings may be totally useless, for instance if $\alpha(\iota) = \iota$ for all α and ι; or if the α-s are simply mixing $\mathcal{S}_{cl}$. In that case nothing useful happens to the classical system that could be used for learning about the quantum one. Entropy of the quantum system is growing with no useful gain. There are either no events and no observations at all, or the events are chaotic and information is being lost. We have, in such a case, a useless dissipation in the quantum system (unless our aim is to build a quantum driven random number generator, which is also of intertest). On the contrary to this, *the aim of any well posed measurement is to get a maximum of information and to pay for it with a minimum of dissipation.*
Because dissipation of quantum states is already fixed in our simplified model by Eq. (7) (independently of the choice of involutive transformations α), it is clear that $\mathcal{S}_{cl}$ and the maps associated to α-s should be chosen in an optimal way. We will see a particular example in the next section, where we will formulate the multi-particle cloud chamber model.[6]

Till now we were discussing time evolution of statistical states. But, as it was already pointed out above, in many experiments we are interested in time series of events. In fact, often these are the only results that are provided. Sometimes (but not always) the experiment can be repeated another time in another place. But even then, this is already another experiment. Fortunately it so happens that the information contained in the Liouville generator on the rhs of Eq. (4) is sufficient for reconstruction of the minimal, piecewise deterministic random process that describes individual histories and which allows for a computer simulation of actual experiments. Although a fully satisfactory mathematical justification of this reconstruction is yet to be given,

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[6]In the following we will always assume that each α acts in a non–trivial way on $\mathcal{S}_{cl}$. Without this assumption the following discussion of the stochastic process would have to be more subtle.

a general idea and examples have been described in Refs. (16-21). We will not repeat the derivation once more. Instead, in the following subsection, we will describe explicitly the piecewise deterministic random process (PDP) that corresponds to the Eq. (4), that is which leads to this equation after averaging over individual histories.

Remark 4 An easy introduction to the theory of PDP that does not require the theory of stochastic differential equations can be found in Refs. (29, 30). Random processes that lead to equations similar to Eq. (7) were considered by other authors - cf. Refs. $(31-34)$, but they were all concerned only with the the quantum part of evolution. They were not concerned with "events" and with information that can be obtained from time series of events. In Pearle[35], Ghirardi and Pearle[36] (see also Gisin[37] and references therein) stochastic differential equations were used that lead to diffusion processes in Hilbert space. Although reproducing the master equation for the quantum subsystem, these diffusion processes fail to be equivalent to PDP processes when applied to $Q + C$. We consider this as an unsatisfactory feature of the quantum diffusion approach and we conjecture that the minimal PDP described in the following sub-section should be used for simulation of discrete time series of experimental events and accompanying them quantum jumps. An illuminating discussion comparing the two approaches (i.e. piecewise deterministic vs continuous diffusion) in the domain of quantum optics experiments can be found in the paper by Wiseman and Milburn[38]. A theoretical scheme which apparently aims at a description of both approaches (but again, not dealing explicitly with the event space) has been developed by Barchielli and Belavkin[39].

2.1   The Piecewise Deterministic Random Process

Here we will describe the piecewise deterministic random process (PDP) on pure states of the total system $Q+C$ that leads to the Eq.(4) after averaging over paths. It is derived in details in Ref. (20).

Suppose at $t = 0$ the quantum system is described by the state vector $\psi_0 \in \mathcal{H}_q$ and the classical system is in the state ι_0. Then ψ develops according to the equation

$$\psi(t) = \frac{\exp\left(-iH_{\iota_0}t - \frac{\Lambda}{2}t\right)}{\|\exp\left(-iH_{\iota_0}t - \frac{\Lambda}{2}t\right)\|}\psi_0 \tag{8}$$

while C remains at i_0 until jump occurs at some random instant of time t_1.

The time t_1 of the jump is governed by the nonhomogeneous Poisson process that can be described as follows: the probability $P(t, t + \triangle t)$ for the jump to occur in the time interval $(t, t + \triangle t)$, provided it did not occurred

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yet, is given by the formula

$$P(t, t + \triangle t) = 1 - \exp\left(-\int_t^{t+\triangle t} \lambda(\psi(s))\right) ds, \qquad (9)$$

where

$$\lambda(\psi) = (\psi, \Lambda\psi). \qquad (10)$$

When the jump occurs at $t = t_1$, then C jumps from i_0 to one of the states $\alpha(i_0)$, say, to $\alpha_1(i_0)$, while quantum state vector jumps at the same time from its actual value $\psi(t_1)$ to $\psi_1 = F_{\alpha_1}\psi(t_1)/\|F_{\alpha_1}\psi(t_1)\|$, and the process starts again.

The probability p_α of choosing a particular value α is given by

$$p_\alpha = \frac{\|F_\alpha\psi(t_1)\|^2}{\lambda(\psi(t_1))}. \qquad (11)$$

Remark 5 It should be noticed that we never assumed that the different F_α-s commute. What can be expected if they do not commute is instability: instead of stabilizing, the sequence $\alpha_n(i_0)$ covers part of $\mathcal{S}_{cl}$ in a chaotic way.[7]

3 Multiparticle Cloud Chamber Model

As an illustration of the above formalism, in this section we will consider a specific class of physical models, namely non–relativistic cloud chamber models.[8] One such model has been already described[20], but only for one particle systems. The formalism developed in the previous section allows us to discuss here a more general case.

We start with a set E, thought of to be the physical *space*. We suppose E is a measurable space, endowed with some measure. To simplify the notation we will denote the points of E by x, an we will write dx for the corresponding measure. We take $\mathcal{H}_1$ to be the Hilbert space of square integrable functions on E.[9] For instance we can think of $E = R^3$ and $\mathcal{H}_1 = L^2(E, d^3x)$ – see the example below. We define $\mathcal{H}_q$ to be the Fock space over $\mathcal{H}_1$, that is:

$$\mathcal{H}_q = \oplus_{n=0}^{\infty}\mathcal{H}_n, \qquad (12)$$

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[7]Of course the actual behavior depends essentially also on the relation of the two time-scales: the one given by the energy spectrum of ψ, and the other provided by the jump rate function.

[8]The Reader may wish to compare our simple model with a Hamiltonian theory developed by Belavkin and Melsheimer in Ref. (39a). The homogeneous Poissonian sampling law is an assumption qin their paper.

[9]Or, more generally, of square integrable sections of a Hermitian vector bundle over E. This will happen for non–scalar particles.

where

$$\mathcal{H}_n = \otimes^n \mathcal{H}_1. \tag{13}$$

We denote by $\mathcal{H}_q^{\pm}$ the symmetric (Boson) and the antisymmetric (Fermion) subspace of $\mathcal{H}_q$ respectively. We denote by $N(x)$ the number operator density:

$$(N(x)\psi)(x_1, \ldots, x_n) = \sum_{i=1}^{n} \delta(x - x_i)\psi(x_1, \ldots, x_n) \tag{14}$$

where $\delta(x - y)$ is the Dirac measure:

$$\int_E f(x)\delta(x - y)dvol(x) = f(y).$$

Then $\mathcal{H}_q^{\pm}$ are invariant under $N(x)$.

Let now E_d be another measurable set. The points a of E_d will parameterize detectors. E_d can be finite (for instance, just one point), it can be infinite denumerable (e.g. an infinite lattice in R^3) or, if $E = R^3$, E_d can be a lower dimensional submanifold of E (a string, a surphace). For definitness we can think of E_d as an an open subset of E (in particular, we can take $E_d = E$). Generalization to other cases presents no problem. One needs, in general, some measure on E_d. The points $a \in E_d$ will play the role of α-s of the previous section.

For each $a \in E_d$, that is for each detector, let there be given a function $f_a(x)$ on E. The physical interpretation is that $f_a(x)$ describes sensitivity of the detector located at the point a. For instance, if E is a Riemannian manifold, we could take for f_a a Gaussian function of the geodesic distance from a. In that case we would have to provide two parameters (that can depend on a) - the height and the width of the Gaussian function. The height would then be approximately inversely proportional to the square root of the response time of the local counter, while the width - to its spatial extension.

A point limit corresponds to $f_a^2(x) \mapsto \lambda\delta(x - a)$.

We choose now the "properties" to be given by the operators[10]

$$F_a = \int_E f_a(x)N(x)dvol(x). \tag{15}$$

Our classical system is, as in the Ref. (20) a continuous medium of 2-state detectors which, at each point $a \in E_d$, can be in one of their two states: "on–state", represented by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, or "off–state", represented by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. We will consider only those configurations of the detector medium which are "on" at a finite number of points. Thus the space $\mathcal{S}_{cl}$ of pure states of the classical

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[10]This choice is equivalent to the one proposed implicitly by Gisin[40]. It is also the same definition as in Ref. (36). Notice howere change in the notation.

system is, in our case, the same as the class of all finite subsets of E_d.[11] We will use the letter Γ to denote its generic point. Thus each Γ is a finite subset of E_d. It represents state of the detector medium characterized by the fact that the detectors at the points of Γ are "on" while those outside of Γ are "off". The symbol Γ plays just the role of ι of the previous section.

According to our general strategy described before, we consider now the total system, consisting of the quantum system represented by the Fock space $\mathcal{H}_q$ and of the classical system – the detectors. Thus states of the total system are given by families $\{\rho_\Gamma\}_{\Gamma \in \mathcal{S}_{cl}}$ such that $\sum_\Gamma \mathrm{Tr}\,(\rho_\Gamma) = 1$. The symbol $\sum_\Gamma$ has to be understood in a generalized sense: it is sum over a discrete index that is the number of points in Γ, and integral over the different configurations that these points can take in E_d.

What remains to be specified to fix our model, and to apply the results of the previous section, are the transformations $a : \Gamma \longmapsto a(\Gamma)$. In our case, when E_d is a subset of E, there is a natural choice: each a flips the detector state at $x = a$. We can write it also as $a(\Gamma) = \{a\} \triangle \Gamma$, where $\triangle$ denotes the symmetric difference.[12] It is evident now that $a(a(\Gamma)) = \Gamma$, and so the results of the previous section are applicable. It should be noticed that the operators F_a commute with the number operator and with particle permutations. It follows that if the quantum Hamiltonian preserves the particle number and particle statistics, then the PD random process preserves them too.

Remark 6 With real detectors it is realistic to assume that after registering an event, a certain "dead" interval of time must lapse before they can be active again. Moreover, for this time interval the quantum Hamiltonian may change in the region occupied by the detector. It is very easy to include such data into our PDP description. However, the modified process cease to be Markovian. We can make it Markovian again by replacing the space of events $\mathcal{S}_{cl}$ with the space of "histories". Then our framework can be applied again provided we allow that $\mathcal{S}_{cl}$ may be different for each t. That is possible and allowing for such a generalization would not change much our discussion.

3.1   Example: Spontaneous Localization Model

We take the simplest case, that of a passive, homogeneous medium[20] in $E = E_d = R^3$. For the functions f_a we take, as in GRW[41], the Gaussian functions:

$$f_a(x) = \lambda^{1/2} \left(\frac{\alpha}{\pi}\right)^{\frac{3}{2}} exp\left(-\alpha(x-a)^2\right), \qquad (16)$$

where α is the parameter determining the width of the Gaussian function. Let us consider the n-particle subspace of the Bosonic Fock space. We are

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[11]Somewhat more detailed discussion can be found in Ref. (20)

[12]Or, in words, $a(\Gamma) = \Gamma \setminus \{a\}$ if $a \in \Gamma$, and $a(\Gamma) = \Gamma \cup \{a\}$ if $a \notin \Gamma$.

working in position representation, so that pure quantum states are described by symmetric wave functions $\psi(x_1, \ldots, x_n)$, with the normalization

$$\int |\psi(x_1, \ldots, x_n)|^2 dx_1 \ldots dx_n = 1. \tag{17}$$

The operators F_a are easy to compute:

$$(F_a \psi)(x_1, \ldots, x_n) = (f_a(x_1) + \ldots + f_a(x_n)) \, \psi(x_1, \ldots, x_n). \tag{18}$$

By simple Gaussian integration we find then the operator $\Lambda = \int F_a^2 da$. It acts as

$$\Lambda = \lambda(n + 2G), \tag{19}$$

where G is the multiplication operator by the function

$$G(x_1, \ldots, x_n) = \sum_{i<j\leq n} \exp\left(-\frac{\alpha}{4}(x_i - x_j)^2\right). \tag{20}$$

The evolution equation (7) for the effective statistical state of the quantum system is now exactly the same as in Ghirardi, Pearle, Rimini paper[41]. Suppose we have free particle dynamics for the quantum particle. Then, between jumps, the wave function evolves, *up to a normalization factor* according to the modified Schrödinger equation:[13]

$$\frac{d\psi(x_1, \ldots, x_n, t)}{dt} = \left(\frac{i\hbar}{2m} \sum_{i=1}^{n} \triangle_{x_i} - \lambda G\right) \psi(x_1, \ldots, x_n, t). \tag{21}$$

It is seen that the extra term in the Schrödinger equation will dump the wave function at the coinciding points $x_i \approx x_j$. The degree of this dumping will depend on the relation of the parameters α, λ, and on the energy spectrum of the wave function.

The rate of jumps is not constant. The probability for a jump to happen in the infinitesimal time interval $(t, t + dt)$, provided it did not occur yet, is, according to the formula (9):

$$P(t, t + dt) = \lambda(t)dt = \lambda\left(n + (\psi_t, G, \psi_t)\right)dt. \tag{22}$$

Here again the coinciding points contribute to a faster reduction rate.

The probability density of triggering a responce from the detector at some point a, given that the event occurs at time t_1, is given by

$$p_a = \|F_a \psi\|^2 / \lambda(t_1). \tag{23}$$

The presence of mixed terms obscures the analysis of most probable behavior. It will depend on the values of the parameters and on the initial wave function. Our formulas provide the frame for a numerical simulation in those ranges where qualitative analysis would provide no guide.

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[13]We omit the scalar dumping term which cancels out after normalization.

4   Summary and Conclusions

We outlined a general philosphy behind the "Event-Enahanced Formalism of Quantum Theory"[14] and we exemplified it with a multiparticle cloud chamber model. In a particular case what we obtained this way covers the improved spontaneous localization model. Although we get the same time evolution equation for the effective statistical state of the quantum system as Ghirardi, Pearle and Rimini, there are differences too. First of all we see that the time evolution of the quantum state separates only in a particular, simple case. Second, for the random process we obtain the minimal PDP that takes place on pure states of $Q + C$. That process, for the cloud chamber model, is easy to verify experimentally and to simulate numerically (apart of the computer power which is necessary for numerical solutions of multiparticle, multidimensional, time-dependent and non-unitary Schrödinger equation). In a particular case, when the quantum Hamiltonian does not depend on the state of the classical detectors, the quantum evolution separates and our process coincides with the Monte Carlo Wave Function algorithm as described in Refs. (31-34).

One important idea of the event-enhanced formalism, namely that the enhanced quantum theory provides its own interpretation, was not discussed in this paper. It will be described in the forthcoming paper by Ph. Blanchard and the present author[44].

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[14]A recent paper by Landsman[42] may be helpful for comparing our philosophy to other approaches, in particular to the "environment–induced superselection rules" of Zurek[43].

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